Reconstruction of Attractors for Options Implied Volatility with Using Fuzzy Clustering Method

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Abstract: ***On the bases of dynamic system theory and fuzzy clustering methods for attractors of options implied volatility on market indexes S&P 500, NASDAQ 100, S&P 100 was researched. Results of researches stay that entropy of implied volatility fuzzy clustering equal from 0.6477 to 0.7986. Implied volatility attractor structure for market indexes are presented with two main components with rest dispersion in interval from 88.82% to 57.59%.On the bases of reconstruction of options attractor implied volatility method the prognoses method for dynamic of options implied volatility was formulated.***

Keywords: Volatility, implied volatility, fuzzy cluster, option, market index.

1. Introduction

The speedy development of financial markets from begin XXI century and world financial crisis in autumn 2008 year open the serious science and practical problems for creating new financial instruments and their using in practice. For effective using this new financial instruments firstly, deepest theoretical background and, secondly, adequate empirical knowledge for basic and derivative financial instruments is needed.

The turnover growth for the derivatives on the stock exchanges was in 1973 as the innovation jump of science researches F.Black, M.Scholes, R.Merton on fair option pricing [2,3,4]. In 1997 R.Merton and M.Scholes was Nobile laureates on economics for the method of derivative pricing. After understanding option pricing formula for all market participants starts the exponential growth of using derivatives in all ways of economical development [18].

In 1900 L.Bachelier with own science work open the forecasting problem hardness for stock pricing on the basis of Brownian motion [1]. R.Engle and T.Bollerslev in your science works going to the conclusion, that asset volatility forecasting is less hard task in comparing with the basis asset price forecasting [7,8].

For the non-standard understanding of pricing processes and using their volatility for the goal of price forecasting R.Engle was awarded to Nobile prize in 1997 for the method of time series analyses on the bases of ARCH mathematical models. Science researches of R.Engle and T.Bollerslev open the science ways for wide using autoregressive conditional heteroskedastic processes, which created classes of models GARCH, E-GARCH, EWMA and stochastic models with spatial probability functions [9].

2. Method

Main goal of this researches is to overcome the mechanical R.Engle and T.Bollerslev decomposition of volatility, formulation of implied volatility forecasting problem on the basis of dynamic systems theory results, Takens theorem and fuzzy clustering methods; developing and implementation the fuzzy clustering methods for attractors of options implied volatility on market indexes S&P 500, NASDAQ 100, S&P 100.

Options implied volatility (IV) on market indexes S&P 500, NASDAQ 100, S&P 100 is the wide using instrument for the financial risks forecasting. Option implied volatility is calculated on the bases of options spot prices traded on Chicago Board of Exchange. The biggest levels of options implied volatility on the last 20 years was on time LTCM bankruptcy, Internet bubble and in autumn 2008 year in time of USA real estate crisis.

In this connection, many world science schools have deep research in the way of options implied volatility forecasting. S. A. Ross and T. Andersen connect volatility and stock pricing on the open market with public information [10,11]. R.F.Engle and V.K.Ng developed more common GARCH model with parameters interpretation in the connection of positive and negative public information [6]. Results of J. Y.Campbell science school are connected with macro economical news, risk rate of returns and their influence to the assets and derivatives volatility. On the bases of this idea J. Y.Campbell formulate common factors, which are connected with volatility: idiosyncratic, industrial and market [12]. Science results of J.M.Maheu and T. H.McCurdy is the practical jump diffusion GARCH model with two classes of news: normal and non-normal. In this model normal news have a slow influence on the dispersion changing in the pricing process. Non-normal news in this model is the source of volatility jumps of basic assets, derivatives and their implied volatility [13].

But, in common, mechanical volatility decomposition from R.Engle time have not the deep fundamental theoretical background and connected with parametric identification different type GARCH models. From this point of view, this method is not adequate to nature of complexness process dynamics [7,8]. This mechanical decomposition for the analyses of pricing processes not present the complexness for volatility interpretation and in practical use is the unnatural defect for this method.
Method which presented in this article solving two problems:

- there are no any functions for the pricing processes;
- the IV dynamic, in the comparing with GARCH models, have, firstly, deep theoretical backgrounds and, secondly, good visual interpretation from statistical point of view (see Fig.1 – Fig.6).

For the developing method of attractors reconstruction for options implied volatility we using the central F.Takens theorem of dynamical systems on attractors reconstruction on the bases of experimental data [14]. In this case options implied volatility IV on market indexes presented from point of view of F.Takens reconstruction procedure, as this is presented in your article [14]:

$$IV = \{iv(t), iv(t+\tau), iv(t+2\tau), ..., iv(t+(m-1)\tau)\}, \quad (1)$$

were m - size of reconstruction for IV (as proposal for the goal of analyses this size of reconstruction is connected with calendar month; for the interpretation – 1 month and 2 month), τ - time shift (on the bases of implied volatility historical quotes). On the choice of parameters m and τ we have many science results, but on the bases all of this results we not made any background conclusions. Than, when we using central theorem of dynamical systems in any sense very important "… do not loss sense of measure and health of sense" [15].

On the bases of reconstructed IV create matrix $[IV_s]$, $s=1,2$ with last row equal to $T \equiv$ 04 December 2008 year:

$$IV_s = \begin{bmatrix} iv(t) & iv(t-\tau_s) & . & . & . & iv(t-(m-1)\tau_s) \\ . & . & . & . & . & . \\ iv(t_i) & iv(t_i-\tau_s) & . & . & . & iv(t_i-(m-1)\tau_s) \\ . & . & . & . & . & . \\ iv(T) & iv(T-\tau_s) & . & . & . & iv(T-(m-1)\tau_s) \end{bmatrix}, \quad (2)$$

were t – first date of market implied volatility quotes. Parameters τ_s is the time shift from 1 to 2 month. Use matrix (2) as initial data for fuzzy clustering the objects $IV(t_i)$, size of which is equal $(1{:}m)$, in quality $|T-t|$. The goal of fuzzy clustering algorithm for objects $IV(t_i)$ of matrix (2) is the square residuals minimization:

$$f(\Im(IV)) = \sum_{j=}^{K} \sum_{i=1}^{|T-t|} \left| iv_i^{(j)} - c_j \right|^2, \quad (3)$$

were $\left| iv_i^{(j)} - c_j \right|^2$ - distance between reconstructed objects $iv_i^{(j)}$ and fuzzy cluster center C_j. This distance is the indicator of belonging to fuzzy cluster. $\Im(IV) = \{IV(t_i) \mid IV(t_i) \in IV\}$ – fuzzy decomposition number of clustering objects $IV(t_i)$.

The algorithm of fuzzy clustering is connected with minimization of function (3), but we have very small information on common convergence that algorithm. Science researches made by R.J.Hathaway and J.C.Bezdek have the conclusions, that when we use in the algorithm of fuzzy clustering the mahalanobis distance in the middle of fuzzy clusters, the number of algorithm iterations is ended and convergence to stationary point of minimization functions (3) or convergence to stationary point [17].

In absolute numbers of cases the level of belonging objects $IV(t_i)$ to cluster $IV(k)$, $\mu_k(IV(t_i))$, $\forall\ IV(t_i) \in IV_s$, $k = 1, ..., c$ have the measure from 0 to 1. Then, the non whole numbers in this interval is a signal of partial belonging to cluster $IV(k)$.

Matrix $\mu_k(IV(t_i))$ of belonging to cluster $IV(k)$ is the decision result of fuzzy task clustering (3) and have the next properties. Elements of this matrix $\mathrm{M} = [\mu_{i,k}], i = 1, ..., |T-t|, k = 1, ..., c$ have the meaning in interval [0,1]. In this conditions, matrix $\mathrm{M} = [\mu_{i,k}]$ with size $[|T-t| \times c]$ define fuzzy clustering task for objects $IV(t_i)$ with restrictions to level of belonging $\mu_{i,k}$ to cluster $IV(k)$:

$$\mu_{i,k} \in [0,1], \quad 1 \le i \le, |\mathrm{T - t}|, \quad 1 \le k \le c, \quad (4)$$

$$\sum_{k=1}^{c} \mu_{i,k} = 1, \ 1 \le i \le |T-t|, \quad (5)$$

$$0 < \sum_{i=1}^{|T-t|} \mu_{i,k} < |T-t|, \ 1 \le k \le c, \quad (6)$$

Result of fuzzy clustering for the number $|T-t|$ objects $IV(t_i)$ defined by next condition:

$$\mathrm{M} = R^{|T-t| \times c} \mid \mu_{i,k} \in [0,1], \forall i,k;$$

$$\sum_{k=1}^{c} \mu_{i,k} = 1, \forall i; \qquad (7)$$

$$0 < \sum_{i=1}^{|T-t|} \mu_{i,k} < |T-t|, \forall k$$

Number of implied volatility fuzzy clusters c from formulas (4), (5), (6) and (7) is uncertainty variable, than for decision the fuzzy task clustering IV - (3), this number of clusters should be define as initial data.

In the conclusion, the formulation of adequacy problem for the decomposition $\Im(IV) = \{IV(t_i) \mid IV(t_i) \in IV\}$

on fuzzy clusters would be next: how much is adequacy for decomposition $\Im(IV)$ - (3) on fuzzy clusters $IV(k)$ with restrictions (4), (5), (6) and (7) and this adequacy should be optimal in any sense to initial data (2).

3. Results of Attractors Reconstruction

In common, the algorithm of fuzzy clustering IV presented in article find decomposition, for defined number of clusters that have optimal parameterization to initial data (2). But from practical point of view optimal parameterization may be have not the best economical interpretation. For defining the adequacy to results of fuzzy clustering decomposition we use the statistical validity for decomposition IV on fuzzy clusters: Partition Coefficient (PC), Classification Entropy (CE), Partition Index (SC), Separation Index (S), Xie and Beni Index (XB), Dunn Index (DI), Alternative Dunn Index (ADI) [17]. Testing for fuzzy clustering method (3) with restrictions (4)- (7) was made for option implied volatility on market indexes S&P 500 (from 2 January 1990), NASDAQ 100 (from 10 October 2000), S&P 100 (from 2 January 1986) to 4-th December 2008. In Table 1 presented statistical validity results of implied volatility clustering.

From Table 1 we may seeing, that classification entropy of IV fuzzy clusters is in interval from 0.6477 to 0.7986. Those values confirm us, that big information volume may be received on the background of statistical valid fuzzy clusters. The conformation for that conclusion is the presentation of IV fuzzy clusters on market indexes (see. Fig.1 – Fig.6). From analyses Fig.1 – Fig.6 we may conclude, that IV options dynamic on market indexes S&P 500, NASDAQ 100, S&P 100 changing on the sustainable configuration – attractors, which presented by transformation with only two components for m -size vector $IV\ (t_i)$ [14,15]. Statistical adequacy for that transformation on the bases the rest dispersion is the next: minimum – 57.59% (see. Fig.6; market index S&P 100, τ_2–2 month), and maximum - 88.82% (see. Fig.3; market index NASDAQ 100, τ_1 – 1 month). For addition, and this is equal to real situation on the market, on Fig, 1, Fig, 2, Fig, 5, Fig, 6 we may see zones with highest risks, which defined by options pricing on market indexes. In the south-west directions on that figures we see the formation new sustainable configuration, which presented new level of risks on the markets.

4. Conclusion

On the background the dynamic system theory, fuzzy clustering methods in this article is formulated and resolved optimization tasks (3) of attractors reconstruction for the options implied volatility on market indexes S&P 500, NASDAQ 100, S&P 100. With using statistical measure of validity was defined, that minimal IV clustering entropy equal to 0.6477, and maximal - 0.7986. Minimal and maximal rest dispersion of attractors reconstruction for the options implied volatility on market indexes S&P 500, NASDAQ 100, S&P 100 and their presentation by only two components equal to 57.59% and 88.82%.

At the end, attractors IV for the options on market indexes S&P 500, NASDAQ 100, S&P 100 and their interpretation with high statistical measure of reconstruction may be use, firstly, for forecasting IV pricing processes on the background serious theoretical bases, and secondly, pricing spot and futures financial risks, which are the main component for economical development.

This method of attractor reconstruction for the options IV on market indexes S&P 500, NASDAQ 100, S&P 100 overcome negative characteristics of mechanistic decomposition for complicated dynamics in pricing processes, with using wide classes ARCH, GARCH methods.

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Figures

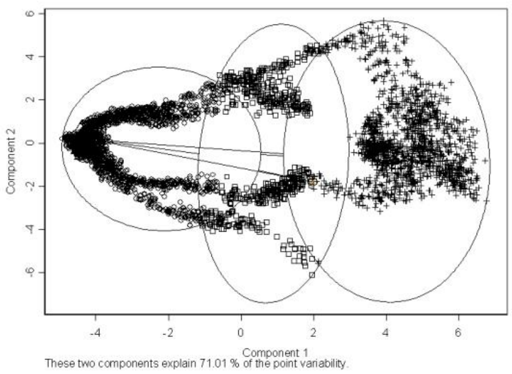

Figure 1: Attractor IV for options on market index S&P 500. τ_1 – 1 month

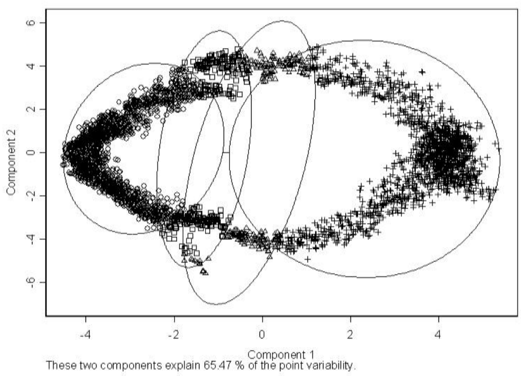

Figure 2: Attractor IV for options on market index S&P 500. τ_2 – 2 month

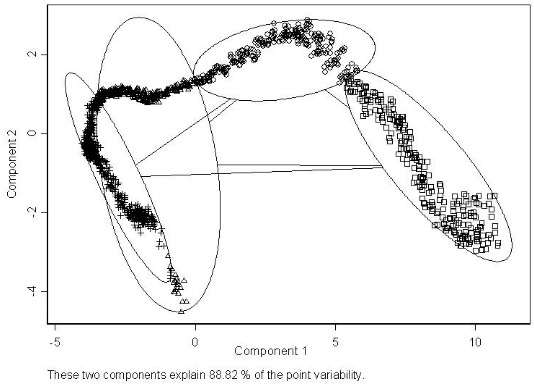

Figure 3: Attractor IV for options on market index NASDAQ 100. τ_1 – 1 month

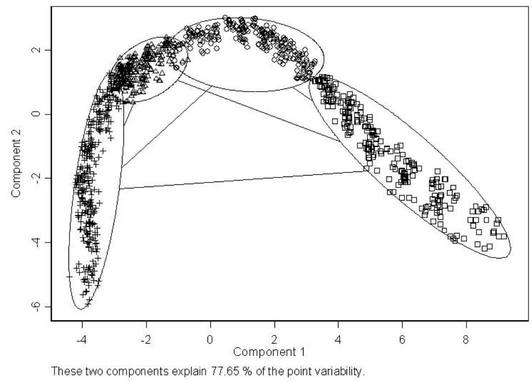

Figure 4: Attractor IV for options on market index NASDAQ 100. τ_2 –2 month

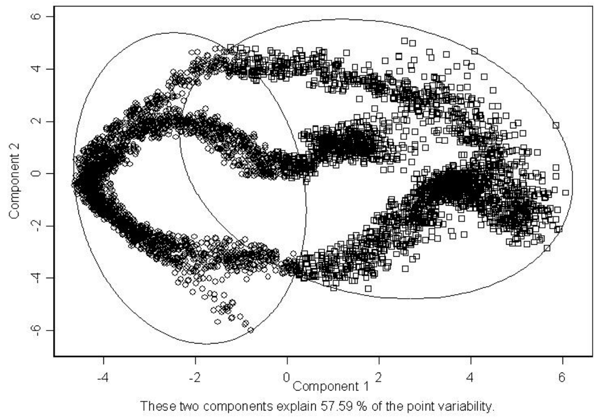

Figure 5: Attractor IV for options on market index S&P 100. τ_1 – 1 month

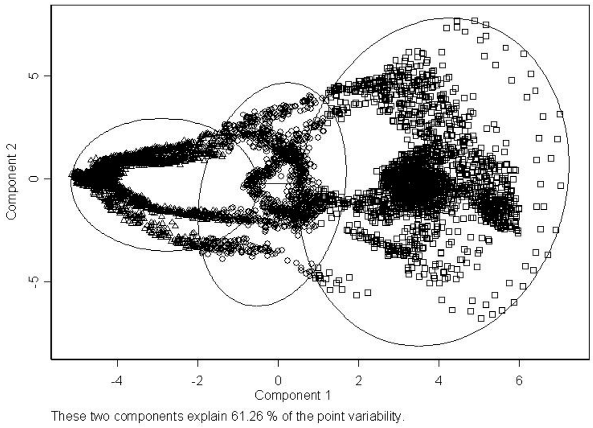

Figure 6: Attractor IV for options on market index S&P 100, τ_2 –2 month

Table 1: Statistical validity of implied volatility clusters

Number of valid clusters	IV options (S&P 500. τ_1 – 1 month) / **3**	IV options (S&P 500. τ_2–2 month)/ **2**	IV options (S&P 100. τ_1 – 1 month) / **3**
Partition Coefficient	0.6292	0.6792	0.6292
Classification Entropy	0.7052	0.724	0.7052
Partition Index	3.1521	4.273	3.1521
Separation Index	0.001	0.0014	0.001
Xie and Beni Index	27.7613	20.7979	27.7613
Dunn Index	0.0005	0.0007	0.0005
Alternative Dunn Index	0.0004	0.0002	0.0004
Number of valid clusters	IV options (S&P 100. τ_2–2 month) / **2**	IV options (NASDAQ 100. τ_1 – 1 month) / **4**	IV options (NASDAQ 100. τ_2–2 month) / **4**
Partition Coefficient	0.6792	0.6621	0.6619
Classification Entropy	0.724	0.654	0.6477
Partition Index	4.273	0.6509	0.4468
Separation Index	0.0014	0.0006	0.0007
Xie and Beni Index	20.7979	17.4487	9.7507
Dunn Index	0.0007	0.0035	0.0041
Alternative Dunn Index	0.0002	0.0024	0. 0012