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Mucus Transport in the Human Lung Airways: Effect of Porosity Parameter and Air Velocity

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Abstract: ***In this paper, a planar two layer steady state fluid model is proposed to study the mucus transport in the human lung airways by taking the effect of air-velocity due to air-motion and porosity parameter due to forming of porous cilia bed by certain immotile cilia by considering mucus as a visco-elastic fluid. It is shown that mucus transport increases as the pressure drop, air velocity due to air-motion increase. It is also observed that mucus transport decreases as the viscosity of serous layer fluid or that of mucus increases, but any increase in mucus viscosity at its larger values does not seem to affect the mucus transport. It is also found that for given total depth of serous layer and mucus layer, there exists a serous fluid layer thickness for which mucus transport is maximum. It is also shown that mucus transport decreases as its elastic modulus increases.***

Keywords: *Mucus transport, porosity parameter, visco-elastic mucus, air velocity.*

1. Introduction

The muco-ciliary system is one of the most important primary defense mechanisms of the human lung airways for cleaning the inspired air of contaminants and for removing entrapped particles such as bacteria, viruses, cellular debris, carcinogens in tobacco smoke, etc. from the lungs through mucus transport. It consists of three layers namely: a mucus layer, a serous layer and the cilia which are small hair-like projections lining with the epithelium of the bronchial respiratory tracts. The serous layer fluid is considered as a Newtonian fluid while mucus as a visco-elastic fluid. It has been pointed out that, in general, mucus transport depends upon the structure of cilia, the functions imparted by cilia tips in the serous sub-layer fluid, the thicknesses and the viscosities of the serous fluid and mucus, the interaction of mucus with the serous layer fluid alongwith the visco-elastic parameter of modulus i.e. elastic modulus of mucus.

In recent decades, the mucus transport in the human lungs has been studied by several researchers. In particular, Barton and Raynor [3] presented an analytical model for mucus transport by considering cilium as an oscillating cylinder with a greater height during the effective stroke and smaller height during the recovery stroke. Blake [5] considered a two-layer Newtonian fluid model, one serous layer fluid and the other mucus and pointed out the importance of gravity and effect of air flow on mucus transport. Another mathematical analysis of two-layer fluid model is given by Blake and Winet [6]. They suggested that if cilia just penetrate the upper, much more viscous layer, then the mucus transport rate would be substantially enhanced.

Though the air flow resistance in bronchial airways has been studied by Schroter and Sudlow [17] and Pedley et al.[13], the role of mucus interaction with mucus in bronchial clearance has been emphasized by Clarke [7], Clarke et al. [8] and many others, including Puchelle et al. [14], Zahm et al. [22], King et al. [9,10,11] in their experimental studies. Scherer and Burtz [16] conducted experiments relevant to coughing and showed the importance of viscosity of the fluid. King et al. [12] also studied the interaction of airflow with the mucus gels in a simulated cough machine under study state and oscillatory airflow conditions and pointed out the importance of viscosity of mucus gel on transport. Agarwal et al.[2] have studied the mucus transport by airflow interaction in a miniaturized simulated cough machine and found that mucus transport increases as the viscosity of the serous layer simulant decreases or as the mucus filance (spinnability) decreases.

King et al. [12] have presented a planar two-layer fluid model for muco-ciliary transport in the respiratory tract due to cilia beating and air motion by considering mucus as visco-elastic fluid and have shown that mucus transport increases as shear stress due to air motion, pressure drop and mean velocity of cilia tips increase. They have also shown that mucus transport rate is maximum at some value of serous fluid thickness for fixed total depth of serous layer fluid and mucus.

Agarwal and Verma [1] presented a two layer steady state mathematical model to study the mucus transport in the respiratory tract due to air-flow by considering cilia bed as porous matrix. In their paper, the effect of air motion is incorporated by prescribing shear stress at the mucus-air interface. They showed that mucus transport increases as the pressure drop, shear stress due to air motion and porosity parameter increase. It was also observed that mucus transport decreases as the viscosity of serous layer or that of mucus increases, but at higher values of mucus viscosity does not affect mucus transport. It was also found that for a fixed total thickness of mucus and serous layer, there exists a serous layer thickness for which the mucus transport is maximum.

In this paper, a two layer steady state mathematical model is presented to study the mucus transport in the human lung airways by taking the following aspects into account:

(i) The serous layer fluid is considered as incompressible Newtonian fluid while mucus layer is considered as a visco-elastic fluid.
(ii) The serous layer fluid is divided into two sub-layers, one in contact with the epithelium and the other in contact with the mucus. It is assumed that cilia are immotile and form a

porous matrix bed in serous sub-layer, where flow may occur due to pressure gradient as considered by Beavers and Joseph [4] and due to porosity parameter as considered by Verma [21]. No net flow is assumed in the serous sub layer in contact with the epithelium.

(iii) The effect of air – motion is incorporated by prescribing air-velocity at the mucus-air interface as a boundary condition.

(iv) The effects of pressure gradients and gravity are also incorporated in the model.

2. Mathematical Model

The physical situation of the transport of serous fluid and mucus in the human lung airways may be represented by a planar two-layer fluid model as shown in Fig.1:

In the serous sub-layer $0 \leq y \leq h_e$, no net flow of the fluid is assumed. However, in the serous sub-layer $h_e \leq y \leq h_s$ and in the mucus layer $h_s \leq y \leq h_m$, the flow of respective fluids is governed by the interaction of cilia, air- motion in contact with the mucus, pressure gradient present in the fluid layers and acceleration due to gravity.

The equations governing the motion of the serous layer fluid and the mucus under steady state and low Reynold's number flow approximations by taking the effect of acceleration due to gravity in the direction of flow, can be written as follows (Blake [5], King et al. [12]):

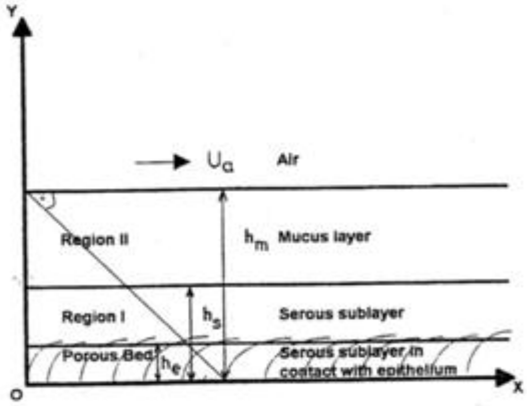

Figure 1: Mucus transport in the human lung airways

Region –I: Serous layer ($h_e \leq y \leq h_s$):

$$\mu_s \frac{\partial^2 u_s}{\partial y^2} = \frac{\partial p}{\partial x} - \rho_s g \cos\alpha \qquad \textbf{(1)}$$

Region-II: Mucus layer ($h_s \leq y \leq h_m$):

$$\frac{\partial \tau_m}{\partial y} = \frac{\partial p}{\partial x} - \rho_m g \cos\alpha \qquad \textbf{(2)}$$

$$\mu_m \frac{\partial u_m}{\partial y} = \tau_m \left[1 + \lambda^2 \left(\frac{\partial u_m}{\partial y}\right)^2\right] \qquad \textbf{(3)}$$

where p is the pressure that is constant across the layers; u_s and u_m are the velocity components of serous sub-layer fluid and mucus in x- direction respectively; ρ_s, μ_s, ρ_m and μ_m are their respective densities and viscosities; g is the acceleration due to gravity and α is the angle by which the airway in the human lungs is inclined with the vertical. Here, h_e is the mean thickness measured from the surface of the epithelium to the tips of cilia during beating i.e. the interface between the two serous sub-layers; h_s is the thickness measured from the surface of the epithelium to the interface between serous sub-layer and mucus and h_m is the thickness measured from the surface of the epithelium to the mucus air-interface, λ ($=\frac{\mu_m}{G}$) is the relaxation time, G is the elastic modulus of mucus and τ_m is the shear stress in the mucus layer. Equation (3) gives the relationships between the shear stress and velocity gradient for visco-elastic fluid in the case of one dimensional flow (Tanner [18], King et al. [12]).

The following boundary and matching conditions are taken for the system of equations (1) - (3)**:**

Boundary Conditions

$$u_s = \beta \frac{\partial u_s}{\partial y}, \; y = h_e \qquad \textbf{(4)}$$

$$u_m = U_a \;, y = h_m \qquad \textbf{(5)}$$

where β is the porosity parameter due to formation of porous matrix bed by immotile cilia in the serous sub- layer in contact with epithelium. The condition (5) implies that the air-velocity is continuous at the mucus- air interface and incorporates the effect of air- motion similar to the analysis of Blake [5].

Matching Conditions

$$u_s = u_m = U_1 \;, y = h_s \qquad \textbf{(6)}$$

$$\mu_s \frac{\partial u_s}{\partial y} = \tau_m \;, y = h_s \qquad \textbf{(7)}$$

where U_1 is the mucus-serous sub layer interface velocity to be determined by using equation(7). The conditions (6) and (7) imply that the velocities and the shear stresses are continuous at mucus-serous layer interface.

3. Analytical Solution

Solving (1)-(3) and using boundary and matching conditions (4)-(7), we get

$$u_s = \frac{\phi_s}{2\mu_s}\left[y^2 - \frac{(h_s^2 - h_e^2 + 2\beta h_e)}{(\beta + h_s - h_e)} y - \frac{\{\beta h_s + (h_s - h_e)h_e\}}{(\beta + h_s - h_e)} h_s\right] + U_1 \frac{(\beta + y - h_e)}{(\beta + h_s - h_e)} \qquad \textbf{(8)}$$

and

$$u_m = \frac{\phi_m}{2\mu_m}(y - h_m)(y - h_s) + \frac{\phi_m^{\;3}}{64\mu_m G^2}\left[\{(y - h_m) + (y - h_s)\}^4 - (h_m - h_s)^4\right] + U_a \qquad \textbf{(9)}$$

where

$$U_1 = -\frac{\phi_s}{2\mu_s}(2\beta + h_s - h_e)(h_s - h_e) - \frac{\phi_m}{2\mu_s}(\beta + h_s - h_e)(h_m - h_s) \qquad \textbf{(10)}$$

$$\phi_s = \frac{\partial p}{\partial x} - \rho_s g \cos\alpha \text{ and } \phi_m = \frac{\partial p}{\partial x} - \rho_m g \cos\alpha \qquad \textbf{(11)}$$

The volumetric flow rates i.e. fluxes in the two layers are respectively defined as follows:

$Q_s = \int_{h_e}^{h_s} u_s dy$ and $Q_m = \int_{h_s}^{h_m} u_m dy$

which after using (8) and (9) are found as:

$$Q_s = -\frac{\phi_s}{3\mu_s}\frac{(h_s - h_e)^4}{(\beta + h_s - h_e)} - \frac{\phi_s}{6\mu_s}\frac{\beta(h_s - h_e)}{(\beta + h_s - h_e)}\{8(h_s - h_e)^2 + 6\beta(h_s - h_e) + 3h_e h_s\} - \frac{\phi_m}{4\mu_s}(2\beta + h_s - h_e)(h_m - h_s)(h_s - h_e) \qquad \textbf{(12)}$$

and

$$Q_m = -\frac{{\phi_m}^3}{80\mu_m G^2}(h_m-h_s)^5 - \frac{\phi_m}{12\mu_m}(h_m-h_s)^3 - \frac{\phi_m}{4\mu_s}(\beta+h_s-h_e)(h_m-h_s)^2 - \frac{\phi_s}{4\mu_s}(2\beta+h_s-h_e)(h_s-h_e)(h_m-h_s) + \frac{U_a}{2}(h_m-h_s) \quad \textbf{(13)}$$

It can be seen by using equation of fluid continuity that Q_s and Q_m are constants, therefore, from equations (12) and (13), we note that $(-\frac{\partial p}{\partial x})$ is also constant. Hence, replacing it by the pressure drop over the mean length L of the cilia forming porous matrix bed, the expressions for the fluxes may be written as:

$$Q_s = \frac{\phi_{s0}}{3\mu_s}\frac{(h_s-h_e)^4}{(\beta+h_s-h_e)} + \frac{\phi_{s0}}{6\mu_s}\frac{\beta(h_s-h_e)}{(\beta+h_s-h_e)}\{8(h_s-h_e)^2+6\beta(h_s-h_e)+3h_e h_s\} + \frac{\phi_{m0}}{4\mu_s}(2\beta+h_s-h_e)(h_m-h_s)(h_s-h_e) \quad \textbf{(14)}$$

and

$$Q_m = \frac{{\phi_{m0}}^3}{80\mu_m G^2}(h_m-h_s)^5 + \frac{\phi_{m0}}{12\mu_m}(h_m-h_s)^3 + \frac{\phi_{m0}}{4\mu_s}(\beta+h_s-h_e)(h_m-h_s)^2 + \frac{\phi_{s0}}{4\mu_s}(2\beta+h_s-h_e)(h_s-h_e)(h_m-h_s) + \frac{U_a}{2}(h_m-h_s) \quad \textbf{(15)}$$

$$\phi_{s_0} = (\frac{\Delta p}{L}+\rho_s g\cos\alpha) \text{ and } \phi_{m_0} = (\frac{\Delta p}{L}+\rho_m g\cos\alpha) \quad \textbf{(16)}$$

where $\Delta p = p_0 - p_L$, $p = p_0$ at x = 0, $\Delta p = p_L$ at $x = L$. It is noted that the effect of acceleration due to gravity is similar to that of the pressure drop.

Now, when $\phi_{m_0} = 0$ and $\phi_{s_0} = 0$, then expressions for volumetric flow rates become:

$$Q_s = 0 \quad \textbf{(17)}$$

$$Q_m = \frac{U_a}{2}(h_m-h_s) \quad \textbf{(18)}$$

Again, when $U_a = 0$, the expressions for volumetric flow rates become:

$$Q_s = \frac{\phi_{s0}}{3\mu_s}\frac{(h_s-h_e)^4}{(\beta+h_s-h_e)} + \frac{\phi_{s0}}{6\mu_s}\frac{\beta(h_s-h_e)}{(\beta+h_s-h_e)}\{8(h_s-h_e)^2+6\beta(h_s-h_e)+3h_e h_s\} + \frac{\phi_{m0}}{4\mu_s}(2\beta+h_s-h_e)(h_m-h_s)(h_s-h_e) \quad \textbf{(19)}$$

$$Q_m = \frac{{\phi_{m0}}^3}{80\mu_m G^2}(h_m-h_s)^5 + \frac{\phi_{m0}}{12\mu_m}(h_m-h_s)^3 + \frac{\phi_{m0}}{4\mu_s}(\beta+h_s-h_e)(h_m-h_s)^2 + \frac{\phi_{s0}}{4\mu_s}(2\beta+h_s-h_e)(h_s-h_e)(h_m-h_s) \quad \textbf{(20)}$$

Remarks: The following remarks can be made by close observation of equations (14)-(15) and (17)-(20) regarding transport rates Q_s and Q_m :

1) From equations (15) and (20), we note that the effect of G on mucus transport is dependent on U_a, ϕ_{m0} and ϕ_{s0}. When these quantities are zero, Q_m does not depend on either G or μ_m. In general, when $U_a \neq 0$, $\phi_{s0} \neq 0$ and $\phi_{m0} \neq 0$, then mucus transport decreases as its elastic modulus G increases.
2) When $\phi_{s0} = 0$ and $\phi_{m0} = 0$, then from (17), we observe that $Q_s = 0$ and from (18), we observe that Q_m increases as the air velocity U_a at the mucus-air interface increases. It is also seen that Q_m increases as the mucus thickness increases. The mucus transport remains relatively independent of mucus viscosity, implying that mucus moves as an elastic slab which is in the line with the findings of Ross and Corrsin [16].
3) When $U_a = 0$, i.e. in the absence of air motion, from equations (19)-(20), we clearly note that Q_s and Q_m both increase as the pressure drop, acceleration due to gravity and porosity parameter increase. It is also seen in this case, that mucus transport decreases as its elastic modulus G increases. Further, in this case, Q_s and Q_m decrease as the viscosities of mucus and serous layers increase.
4) Also, when $h_s \rightarrow h_e$ i.e. for negligible thickness of serous layer, then from (19), we get $Q_s = 0$ and from (20), we note that Q_m increases as the mucus thickness increases. Also, for $\phi_{m0} = 0$, Q_m does not depend on mucus viscosity. However, for $\phi_{m0} > 0$, Q_m decreases as mucus viscosity increases. This particular case corresponds with experimental studies of King et al. [09, 10]. The predictions of the mathematical model are in general agreement with those obtained experimentally, i.e. positive dependence on viscosity in the absence of serous layer.
5) To see the effect of mucus thickness on mucus transport in the general case, we find from (15) the rate of change of Q_m for a fixed total thickness of mucus and serous layers as follows:

$$\frac{\partial Q_m}{\partial h_s} = \frac{\phi_{s0}}{4\mu_s}\{2(\beta+h_s-h_e)(h_m-h_s) - (2\beta+h_s-h_e)(h_s-h_e)\} + \frac{\phi_{m0}}{4\mu_s}\{2(\beta+h_s-h_e)+(h_m-h_s)\}(h_m-h_s) - \frac{\phi_{m0}}{4\mu_m}\left[1+\left\{\frac{\phi_{m0}}{2G}(h_m-h_s)\right\}^2\right](h_m-h_s)^2 - \frac{U_a}{2} \quad \textbf{(21)}$$

From equation (21), we note that $\frac{\partial Q_m}{\partial h_s}$ can be negative, zero or positive depending on the value of h_s and other parameters. This implies that there may exist a critical value h_0 of h_s for which Q_m may be maximum. Thus, for fixed total thickness of mucus and serous layers and for some values of $h_s > h_0$, mucus transport may increase with decreasing serous layer thickness (i.e., with increasing mucus thickness), while for the other values of $h_s < h_0$, mucus transport may decrease with decreasing thickness of the serous layer. The former result is in line with the experimental observations of King et al. [10] as pointed out earlier, while the latter result is similar to that obtained by Ross and Corrsin [15].

6) From equation (15), we notice that the coefficient of $\frac{1}{G^2}$ is always positive; hence, the mucus transport Q_m increases as G decreases for given values of various parameters. This implies that mucus transport increases as its elastic modulus decreases in the general case also. The former result is in line with the experimental observations of

King et al. [09,10] for mucus gel simulants. Similar results have been obtained by Verma and Tripathee [20].

4. Results and Discussion

To study the effect of various parameters on mucus transport rate quantitatively, the expression for Q_m given by (15) can be written in non-dimensional form as:

$$\bar{Q}_m = \frac{\bar{\phi}_{m0}^3}{80\bar{\mu}_m \bar{G}^2}(1-\bar{h}_s)^5 + \frac{\bar{\phi}_{m0}}{12\bar{\mu}_m}(1-\bar{h}_s)^3 + \frac{\bar{\phi}_{m0}}{4\bar{\mu}_s}(\bar{\beta}+\bar{h}_s-\bar{h}_e)(1-\bar{h}_s)^2 + \frac{\bar{\phi}_{s0}}{4\bar{\mu}_s}(2\bar{\beta}+\bar{h}_s-\bar{h}_e)(\bar{h}_s-\bar{h}_e)(1-\bar{h}_s) + \frac{\bar{U}_a}{2}(1-\bar{h}_s) \quad \textbf{(22)}$$

by using the following non-dimensional parameters:

$$\bar{\beta} = \frac{\beta}{h_m}, \bar{h}_e = \frac{h_e}{h_m}, \bar{h}_s = \frac{h_s}{h_m}, \bar{\mu}_s = \frac{\mu_s}{\mu_0}, \bar{\mu}_m = \frac{\mu_m}{\mu_0}, \bar{\phi}_{s0} = \frac{\phi_{s0}h_m^2}{\mu_0 U_0}, \bar{\phi}_{m0} = \frac{\phi_{m0}h_m^2}{\mu_0 U_0}, \bar{U}_a = \frac{U_a}{U_0}, \bar{G} = \frac{G h_m}{\mu_0 U_0}, \bar{\lambda}_0 = \frac{1}{\bar{G}}, \bar{Q}_m = \frac{Q_m}{h_m U_0}. \quad \textbf{(23)}$$

where μ_0 is the viscosity of the serous sub layer fluid in contact with epithelium.

Expression for $\bar{Q}_m$ given by (22) is plotted in Fig. 2 to 7 using the following set of parameters which have been calculated by using typical values of various characteristics related to airways (King et al.[12], Agarwal and Verma [1]):

$$\bar{\beta} = 0.02 - 0.10, \bar{h}_e = 0.1, \bar{h}_s = 0.1 - 0.8, \bar{\phi}_{s0} = 1, \bar{\phi}_{m0} = 5 - 20, \bar{\mu}_s = 1 - 10, \bar{\mu}_m = 10 - 100, \bar{U}_a = 0.020 - 0.040, \bar{\lambda}_0 = 0 - 0.10 \quad \textbf{(24)}$$

Fig.2 illustrates that for the fixed values of $\bar{\beta} = 0.02, \bar{h}_e = 0.10$, $\bar{h}_s = 0.20$, $\bar{\phi}_{s0} = 1, \bar{\phi}_{m0} = 5$, $\bar{U}_a = 0.020$, and $\bar{\lambda}_0 = 0.02$, mucus transport decreases as the viscosity of the serous layer fluid or that of the mucus increases. However, increases in mucus viscosity at larger values do not have any significant effect on its transport. This corresponds to the result that mucus moves as an elastic slab (Ross and Corrsin [15] and King et al. [10, 11]).

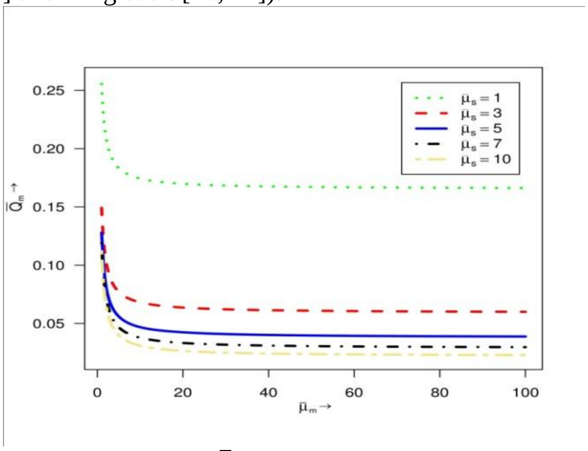

Figure 2: Variation of $\bar{Q}_m$ with $\bar{\mu}_m$ for different values of $\bar{\mu}_s$

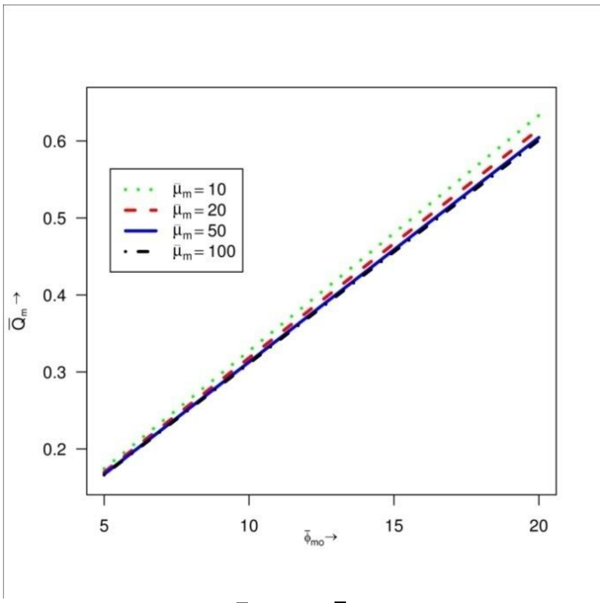

Figure 3: Variation of $\bar{Q}_m$ with $\bar{\phi}_{m0}$ for different values of $\bar{\mu}_m$

Fig.3 illustrates that for the fixed values of $\bar{\beta} = 0.02, \bar{h}_e = 0.10$, $\bar{h}_s = 0.20$, $\bar{\phi}_{s0} = 1, \bar{\phi}_{m0} = 5$, $\bar{U}_a = 0.020$ and $\bar{\lambda}_0 = 0.02$, mucus transport increases as the pressure drop in mucus layer or acceleration due to gravity increases, but it decreases with increase in its viscosity, the relative decrease being larger at larger values of the pressure drop or acceleration due to gravity. This result is in line with the analytical results of Agarwal and Verma [1], Verma [19,21] King et al. [12] and the experimental findings of King et al. [09].

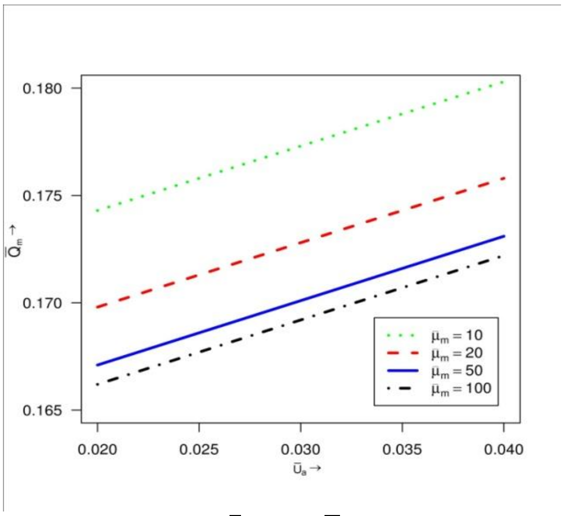

Figure 4: Variation of $\bar{Q}_m$ with $\bar{U}_a$ for different values of $\bar{\mu}_m$

Fig. 4 illustrates that for the fixed values of $\bar{\beta} = 0.02, \bar{h}_e = 0.10$, $\bar{h}_s = 0.20$, $\bar{\mu}_s = 1, \bar{\phi}_{s0} = 1, \bar{\phi}_{m0} = 5$ and $\bar{\lambda}_0 = 0.02$, mucus transport increases as the air-velocity (due to air-motion) at the mucus air-interface increases, but it decreases as its viscosity increases. This is in line with the analytical results of Verma [21].

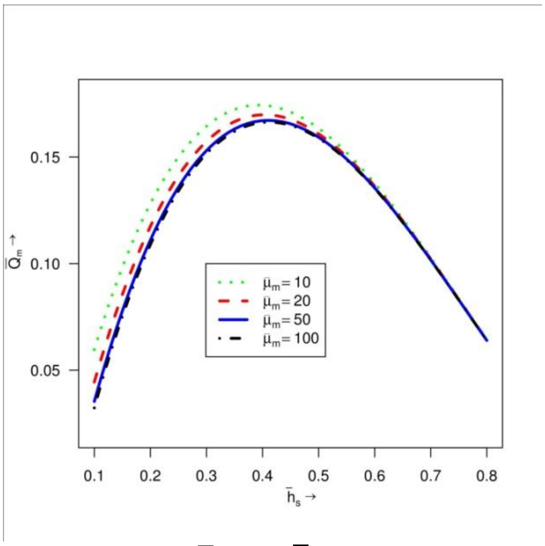

Figure 5: Variation of $\overline{Q}_m$ with $\overline{h}_s$ for different values of $\overline{\mu}_m$

Fig 5 illustrates that for the fixed values of $\bar{\beta} = 0.02, \bar{h}_e = 0.10$, $\bar{\mu}_s = 1, \bar{\phi}_{s0} = 1, \bar{\phi}_{m0} = 5$, $\bar{U}_a = 0.020$ and $\bar{\lambda}_0 = 0.02$, mucus transport increases as $\bar{h}_s$ increases upto a critical values of $\bar{h}_s$ (approximately equal to 0.36) after which it start decreasing with increasing $\bar{h}_s$. Since $\bar{Q}_m$ approaches to unity, this implies that for a fixed total thickness of mucus and serous layer, there exists an optimum value of $\bar{Q}_m$ for some value of serous layer thickness. The conclusion corresponding to decrease in mucus transport with decrease in serous layer thickness is in line with the analysis of Ross and Corrsin [15], Agarwal and Verma [1] and Verma [19].

Fig. 6 shows the variation of $\bar{Q}_m$ with $\bar{\beta}$ for different values of $\bar{\mu}_m$ and $\bar{h}_s$ and fixed values of $\bar{h}_e = 0.1, \bar{\mu}_s = 1, \bar{U}_a = 0.02, \bar{\phi}_{s_0} = 1, \bar{\phi}_{m_0} = 20$ and $\bar{\lambda}_0 = 0.02$. This figure illustrates that mucus transport rate $\bar{Q}_m$ increases as $\bar{h}_s$ increases upto a maximum value of $\bar{h}_s$ after which it starts decreasing with increasing value of $\bar{h}_s$. This figure also illustrates that the mucus transport decreases as its viscosity increases. It is also clear from the figure that the mucus transport increases as the porosity parameter increases.

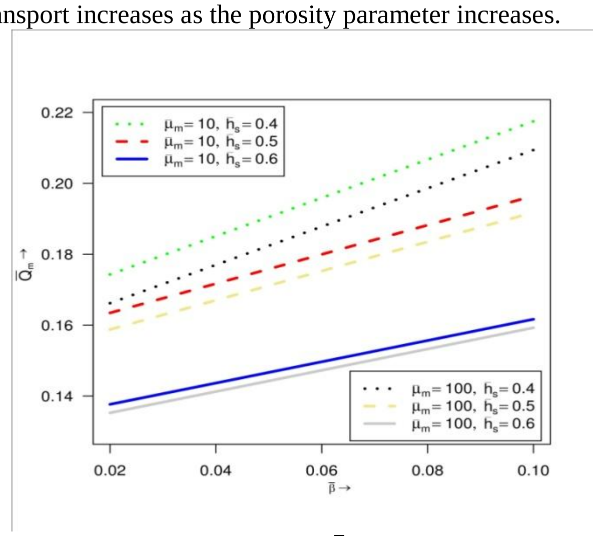

Figure 6: Variation of $\bar{Q}_m$ with $\bar{\beta}$ for different values of $\bar{\mu}_m$ and $\bar{h}_s$

Fig.7 illustrates that for the fixed values of $\bar{\beta} = 0.02, \bar{h}_e = 0.10$, $\bar{\mu}_s = 1, \bar{\phi}_{s0} = 1, \bar{\phi}_{m0} = 5$ and $\bar{U}_a = 0.020$, mucus transport deceases as $\bar{h}_s$ increases or as the mucus viscosity increases. This figure also illustrates that the mucus transport becomes independent with $\overline{\lambda_0}$ for a fixed value of mucus viscosity, from which we conclude that the mucus transport decreases as its elastic modulus increases. This is in line with the analytical results of King et al. [12].

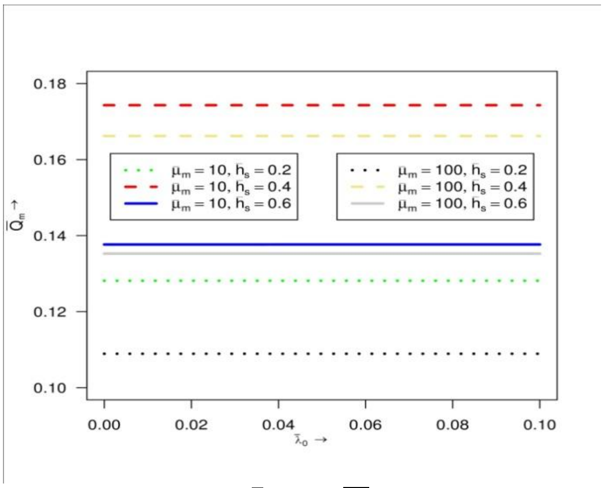

Figure 7: Variation of $\bar{Q}_m$ with $\overline{\lambda_0}$ for different values of $\bar{\mu}_m$ and $\bar{h}_s$.

5. Conclusion

In this paper, we have presented a planar two-layer mathematical model to study mucus transport in the human lung airways under steady state condition by incorporating the effects of porosity parameter due to formation of porous matrix bed by certain immotile cilia and air-velocity due to air-motion by considering mucus as a visco-elastic fluid. The effect of air-motion is considered by prescribing air-velocity at the mucus air interface.

The governing equations of motions are written and have been solved analytically and the effects of various parameters on the mucus transport rate have been discussed. Furthermore, the effects of values of various parameters on mucus transport rate have been computed numerically and have been explained graphically.

It is shown that mucus transport increases as the pressure drop, air velocity due to air-motion and porosity parameter due to formation of porous matrix bed by certain immotile cilia increase. It is also noted that the effect of acceleration due to gravity is similar to that of the pressure drop. It is also observed that mucus transport decreases as the viscosity of serous layer fluid or that of mucus increases, but any increase in mucus viscosity at its larger values does not seem to affect the mucus transport. It is also found that for given total depth of serous layer and mucus, there exists a serous fluid layer thickness for which mucus transport is maximum. It is also seen that mucus transport decreases as its elastic modulus increases.

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