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New Types of Connected Spaces by Semi Cocompact Open Set

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Abstract: ***The aim of this work studies the definitions of certain types of connected spaces by s-coc-open set and we study the relation between them.***

Keyword: s-coc-open set,ω s $-$ coc open set, s-coc-continuous, s-coc′-continuous, s-coc-open function, s-coc-open function, super s-coc open function, s-coc-connected spece, s-coc-locally connected space, s-coc-extremelly disconnected and s-coc-hyper connected space

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1. Introduction

In [6] give definition of connected space. In section one; we defined of s-coc-open set and ωs-coc-open set. In section two we study (s-coc-open, s-coc′-open and super s-coc-open) function. Insection three we defined new types of connected spaces and we study the relation between them.

Definition (1.1) [7]

A subset A of a space $(X,\tau)$is called cocompact open set (coc-open-set) if every $x \in A$ there exists open set $U \subseteq X$ and compact subset K such that $x \in U - K \subseteq A$, the complement of coc-open set is called coc-closed set.

Definition (1.2) [10]

A subset A of space X is called semi open set (s-open) if and only if $A \subseteq \overline{A^{\circ}}$ and A called s-closed if and only if A^{c} s-open.

Proposition (1.1)[9]

For any subset A of space X the following statements are equivalent.

1. $A$is s-open set.
2. $\overline{A} = \overline{A^{\circ}}$
3. There exists open set G such that $G \subseteq A \subseteq \overline{G}$

Remark (1.1) [10]

Every open set is semi-open. But the convers is not true.

Proposition (1.2) [3]

For any subset A of a space X the following statements are equivalent

1. A is s-closed
2. $A^{\circ} = \overline{A}^{\circ}$
3. There exists closed set F in Xsuch that $F^{\circ} \subseteq A \subseteq F$

Definition (1.3)

A subset A of a space (X,τ) is called semi cocompact open set (s-coc-open-set) if for every $x \in A$ there exists s-open set $U \subseteq x$ and compact subset K such that $x \in U - K \subseteq A$, the complement of s-coc-open set is called s-coc-closed set.

Remark (1.2)

Every coc-open set is s-coc-open set. But the convers is not true for the following example:

Example (1.1)

Let $X = \{1,2,3,4,\dots.\}, \tau = \{\emptyset, X, \{2\}, \{3\}, \{2,3\}\}$ topology on$X$, $A = \{1,2\}$ s-coc-open set but not coc-open set.

Remark (1.3):

i- Every open set is s-coc-open set.
ii- Every s-open set is s-coc-open set proof:

Proof

i. Let A open set. Then A s-open and Kcompact. Then for all $x \in A$ we have$x \in A - K \subseteq A$.
ii. Clear.

Remark (1.4):

1. The intersection of open set and s-open is s-open [10].
2. The intersection of two s-coc-open is s-coc-open set.
3. The intersection of s-coc-open and coc-open set is s-coc-open
4. The union of s-coc-open is s-coc-open set
5. The intersection of s-coc-open sets and open set is s-coc-open

Proof:

2. Let A and B s-coc-open sets. To prove $A \cap B$ is s-coc-open set. And let $A \cap B$ is not s- coc- open set. Then there exists $x \in A \cap B$ such that for all V_x s-opet set and K compact $x \in V_x - K \not\subseteq A \cap B$. Then $x \in V_x - K \not\subseteq A$ or$x \in V_x - K \not\subseteq B$. Then $A$is not s-cocor$B$ is not s-coc-

open set. This conduction since A, B s-coc-open sets. Then $A \cap B$ is s-coc-open set.

3. Let A s-coc-open set and B coc-open set. Since B coc-open then B s-coc-ope. Then $A \cap B$ is s-coc-open set by (2)

4. $\{A_\alpha : \alpha \in \Lambda\}$s-coc-open set.let $x \in \cup A_\alpha$. Then$x \in A_\alpha$ for some $\alpha \in \Lambda$.Thenthere exists U_α s-openset and K_α compact such thatx $\in U_\alpha - K_\alpha \subseteq A_\alpha \subseteq \cup A_\alpha$ thenx $\in U_\alpha - K_\alpha \subseteq \cup A_\alpha$. Then$\cup A_\alpha$ s-coc-open set.

5.Let A s-coc-open set and B open set .Then for all $x \in A$ there exists U s-open set and K compact such that $x \in U - K \subseteq A$, since U s-open and B open, then $A \cap B$ s-open by (1), then $x \in (U - K) \cap B \subseteq A \cap B$ thenthenx $\in (U \cap B) - K \subseteq A \cap B$then $A \cap B$ s-coc-open set.

Remark (1.5):

1. The coc-open sets forms topology on X denoted by τ^k[11].
2. The s-coc-open sets forms topology on X denoted byτ^{sk}.
3. Every s-closed is s-coc-closed but the converse is not true for example.

Example (1.2)

Let$X = \{1, 2, 3, 4, \dots\}, \tau = \{\emptyset, X, \{2\}, \{3\}, \{2, 3\}\}$ topology on X and $A = \{1, 2\}$s-coc-open set then$A^c = \{3, 4, 5, 6, \dots\}$ s-closed but A^c not s-closed.

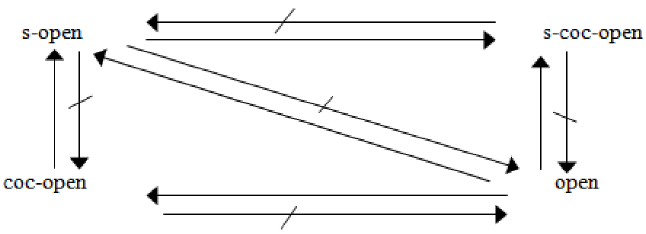
Proposition (1.3)

Let X and Y be topological spaces, and $A \subseteq X, B \subseteq Y$ such that A s-coc-open set in X and B s-coc-open set in Y then $A \times B$ is s-coc-open subset in $X \times Y$.

Proof:

Let $(x, y) \in A \times B$, then $x \in A$ and $y \in B$ .Since A is s-coc-open in X. Then for all $x \in A$ there exists U s-open set and K_1 compact such that $x \in U - K_1 \subseteq A$. Since B is s-coc-open in Y. Then for all $y \in B$ there exists V s-open set and K_2compact such that $y \in V - K_2 \subseteq B$.

Since U and V are s-open sets, Then $U \subseteq \overline{U^\circ}$ and $V \subseteq \overline{V^\circ}$ then $U \times V \subseteq \overline{U^\circ} \times \overline{V^\circ} \subseteq \overline{U^\circ \times V^\circ} \subseteq \overline{(U \times V)^\circ}$.Then $U \times V \subseteq \overline{(U \times V)^\circ}$, then$U \times V$ s-open in $X \times Y$and $K_1 \times K_2$ compact in $X \times Y$ .Then for all $(x, y) \in A \times B$ there exists s-open $U \times V = W$ and $K_1 \times K_2 = K$ compat such that $(x, y) \in W - K \subseteq A \times B$thereforA $\times B$ s-coc-open in $X \times Y$.



Definition (1.4)

Let X be space and $A \subseteq X$. The intersection of all s-coc-closed sets$X$ containing A called the s-coc- closure of A defined by $\overline{A}^{s-coc}$.

$$\overline{A}^{s-coc} = \cap \{B : B \ s\!-\!coc\!-\!closed \ in \ X \ and \ A \subseteq B\}$$

Definition (1.5)[7]

Let X be space and $A \subseteq X$. The intersection of all coc-closed sets X containing A called the coc- closuer of A defined by$\overline{A}^{coc} = \cap \{B : B \ \ coc\!-\!closed \ in \ X \ and \ A \subseteq B\}$.

Proposition (1.4)

Let X be a topological space and $A \subseteq X$then $\overline{A}^{s-coc}$ is the smallest s-coc-closed set containing A.

Proof

Clear.

Proposition (1.5)

Let X be a topological space and$A \subseteq X$, then $x \in \overline{A}^{s-coc}$ if and only if for each s-coc-open in X contained point x we have $U \cap A \neq \emptyset$.

Proof:

Assume that $x \in \overline{A}^{s-coc}$ and let U s-coc-open in X such that $x \in U$, and suppose $U \cap A \neq \emptyset$then $A \subseteq U^c$. Since$U$ s-coc-open set in X and $x \in U$ then U^c s-coc closed set in X and $x \notin U$and $\overline{A}^{s-coc}$ is smallest s-coc-closed containing Athen $\overline{A}^{s-coc} \subseteq U^c$. Since $U \cap U^c = \emptyset$ and $x \in U$ then $x \notin U^c$ then $x \notin \overline{A}^{s-coc}$.
Conversely:

Let U s-coc-closed set in X such that $x \in U$ and $U \cap A \neq \emptyset$.To prove$x \in \overline{A}^{s-coc}$ .Let $x \notin \overline{A}^{s-coc}$ then $x \in (\overline{A}^{s-coc})^c$, since $\overline{A}^{s-coc}$ is s-coc-closed in X, $(\overline{A}^{s-coc})^c$ is s-coc-open in $X$and $\overline{A}^{s-coc} \cap (\overline{A}^{s-coc})^c = \emptyset$ .Then $A \cap (\overline{A}^{s-coc})^c = \emptyset$, since$A \subseteq (\overline{A}^{s-coc})^c$ .This contradiction since every s-coc-open Uin X, $U \cap A \neq \emptyset$.

Proposition (1.6)[7]

Let X be a topological space and $A \subseteq X$, then $x \in \overline{A}^{coc}$ if and only if for each coc-open in X contained point x we have $U \cap A \neq \emptyset$.

Proposition (1.7)

Let X be a topological space and $A \subseteq B$ then

i- $(\overline{A}^{s-coc})^c$ is s-coc-closed set

ii- A is s-coc-closed if and only if $A = \overline{A}^{s-coc}$

iii- $\overline{A}^{s-coc} = \overline{\overline{A}^{s-coc}}^{s-coc}$

iv- If $A \subseteq B$ then $\overline{A}^{s-coc} \subseteq \overline{B}^{s-coc}$

v- $\overline{A}^{s-coc} \subseteq \overline{A}$

vi- $\overline{A}^{s-coc} \subseteq \overline{A}^{coc}$

Proof:

i- By definition of s-coc-closed set.

ii- Let A is s-coc-closed in X. Since $A \subseteq \overline{A}^{s-coc}$ and $\overline{A}^{s-coc}$ smallest s-coc-closed set containing A, then $\overline{A}^{s-coc} \subseteq A$ then $A = \overline{A}^{s-coc}$

Conversely:

Let$A = \overline{A}^{s-coc}$. Since$\overline{A}^{s-coc}$ is s-coc-closed then A is s-coc-closed.

iii- From (i) and (ii)

iv- Let$A \subseteq B$. Since$B \subseteq \bar{B}$ then $A \subseteq \overline{A}^{s-coc}$. Since $\overline{A}^{s-coc}$ smallest s-coc-closed set containing A then $\overline{A}^{s-coc} \subseteq \overline{B}^{s-coc}$

v- By proposition (1.5).

vi-By proposition (1.5) and proposition (1.6).

Definition (1.6)

Let X be space and $A \subseteq X$. The union of all s-coc-open sets of X containing in A is called s-coc-Interior of A denoted by $A^{\circ s-coc} = \cup\{B: B\ s-coc-open\ in\ X\ and\ A \subseteq B\}$

Definition (1.7)[7]

Let X be space and $A \subseteq X$. The union of all coc-open sets of X containing in A is called coc-Interior of A denoted by $A^{\circ coc} = \cup\{B: B\ \ coc-open\ in\ X\ and\ A \subseteq B\}$

Proposition (1.8):

Let X be a topological space and $A \subseteq X$, then $A^{\circ s-coc}$ is the largest s-coc-open set contain A

Proof:

Clear.

Proposition (1.9)

Let X be a topological space and $A \subseteq X$, then $x \in A^{\circ s-coc}$ if and only if there exists s-coc-open set V containing x such that $x \in V \subseteq A$.

Proof:

Let $x \in A^{\circ s-coc}$ then$x \in \cup\, U$ such that U s-coc-open set and $x \in V \subseteq A$.

conversely:

Let there exists V s-coc-open set such that $x \in V \subseteq A$then $x \in \cup\, V, V \subseteq A$ and V s-coc-open set then $x \in A^{\circ s-coc}$.

Proposition (1.10)[7]

Let X be a topological space and $A \subseteq X$, then $x \in A^{\circ coc}$ if and only if there exists coc-open set V containing x such that $x \in V \subseteq A$.

Proposition (1.11)

Let X be a topological space and $A \subseteq B \subseteq X$then.

1. $A^{\circ s-coc}$ is s-coc-open set.
2. A is s-coc-open if and only if $A = A^{\circ s-coc}$.
3. $A^{\circ} \subseteq A^{\circ s-coc}$.
4. $A^{\circ s-coc} = (A^{\circ s-coc})^{\circ s-coc}$.
5. if$A \subseteq B$ then $A^{\circ s-coc} \subseteq B^{\circ s-coc}$.
6. $A^{\circ coc} \subseteq A^{\circ s-coc}$

Proof:

1. and 2. from definition (1.6)
3. Let$x \in A^{\circ s-coc}$ then there exists U s-coc- open set such that $x \in U \subseteq A$ then U s-coc-open set then U s-coc-open set such that $x \in U \subseteq A$ thus $x \in A^{\circ s-coc}$
4. from (1) and (2).
5. Let $x \in A^{\circ s-coc}$ then there exists V open set such that $x \in V \subseteq A$ by proposition (1.9) since $A \subseteq B$ then $x \in V \subseteq B$. Then $x \in B^{\circ s-coc}$.Thus $A^{\circ s-coc} \subseteq B^{\circ s-coc}$.
6. By proposition (1.9) and proposition (1.10).

Proposition (1.12)

Let X be a space and $A \subseteq X$, then $(A^c)^{\circ s-coc} = (\overline{A}^{s-coc})^c$

Proof

Let $x \in (A^c)^{\circ s-coc}$ and $x \notin (\overline{A}^{s-coc})^c$. Then $x \in \overline{A}^{s-coc}$.Then for all $x \in A$ there exists U s-coc-open setsuch that $U \cap A \neq \Phi$. Since $(A^c)^{\circ s-coc}$ s-coc-open set then $(A^c)^{\circ s-coc} \cap A \neq \emptyset$.Then $(A^c)^{\circ s-coc} \subseteq A^c$ then$A \cap A^c \neq \emptyset$.This is contradiction Thus $x \in (\overline{A}^{s-coc})^c$.Then $(A^c)^{\circ s-coc} \subseteq (\overline{A}^{s-coc})^c$. Let $x \in (\overline{A}^{s-coc})^c$then $x \notin \overline{A}^{s-coc}$. Then there exists U s-coc-open set such that $U \cap A \neq \emptyset$. Then $U \subseteq A^c$There for $U^{\circ s-coc} \subseteq (A^c)^{\circ s-coc}$ .Thus $x \in (A^c)^{\circ s-coc}$ then $(A^c)^{\circ s-coc} = (\overline{A}^{s-coc})^c$.

Definition (1.8):[2]

Let X be a space and B any subset of x, a neighborhood (nbd) of B is any subset of X which contains an open set containing BThe neighborhoods of a subset $\{x\}$ is also neighborhood of the point x.

Remark (1.6)

The collection of all neighborhoods of the subset B of X are denoted by $N(B)$. In particular the collection of all neighborhoods of x is denoted by $N(x)$.

Definition (1.9)

Let X be a space andB$\subseteq X$, an s-coc-neighborhood of B is any subset of X which contains an s-coc-open set containing B.The s-coc-neighborhood (s-coc- nbd) of subset $\{x\}$ is also called s-coc-neighborhood of the point x.

Remark (1.7)

The collection of all neighborhoods of the subset B of X are denoted by $N_{s-coc}(B)$ in particular the collection of all neighborhoods of x is denoted by $N_{s-coc}(x)$.

Proposition (1.13)

Let (X, τ) be a topological space and for each $x \in X$, let $N_{s-coc}(x)$ be a collection of all s-coc- neighborhoods of $x$then:

i. If $A \in N_{s-coc}(x)$ such that $A \subseteq B$ then $B \in N_{s-coc}(x)$
ii. If $A, B \in N_{s-coc}(x)$ then $A \cap B \in N_{s-coc}(x)$ such that $A, B \subseteq X$
iii. If $A_\alpha \in N_{s-coc}(x)$ then $\cup A_\alpha \in N_{s-coc}(x)$

Proof

i- Since $A \in N_{s-coc}(x)$ then there exists U s-coc-open set such that $x \in U \subseteq A$, since $A \subseteq B$ then $x \in U \subseteq B$ hence $B \in N_{s-coc}(x)$.

ii-Let $A, B \in N_{s-coc}(x)$ and $A \cap B \notin N_{s-coc}(x)$ .Then $x \in A \cap B$ and for all U s-coc-open set such that $x \in U \nsubseteq A \cap B, x \in U \nsubseteq A or x \in U \nsubseteq B$.Then $A \notin N_{s-coc}(x) or B \notin Ns-coc x$ this contradiction.

iii- Since $A_\alpha \in N_{s-coc}(x)$ exists U_α s-coc-open set such that $x \in U_\alpha \subseteq A_\alpha \subseteq \bigcup A_\alpha$ .Then $x \in U_\alpha \subseteq \bigcup A_\alpha$ .Therefore $\bigcup A_\alpha \in N_{s-coc}(x)$.

Proposition (1.14)

Let (X, τ) be a space and $A \subseteq X$ then A s-coc-open set in X if and only if A is s-coc- neighborhood for all his points in A

Proof

Let A s-coc-open and $x \in A$.Since $x \in A \subseteq A$ then A is s-coc- neighborhood of x for all x hence A is s-coc-neighborhood for all his points

conversely:

Let A is s-coc- neighborhood for all his points and $x \in A$ .Then A is s-coc- neighborhood for x then there exists U_x s-coc-open set such that $x \in U_x \subseteq A$ .Then $A = \bigcup\{x : x \in A\} \subseteq \{U_x : x \in U_x\} \subseteq A$. Then $A = \{U_x : x \in U_x\}$.Then A union of s-coc-open sets .Therefore A is s-coc-open set

Proposition (1.15)

Let X be a topological space and τ discret topology in X then $N_{s-coc}(x) = \{A : x \in A\}$

Proof

Since τ discret topology $then$ $\tau = \{A : x \in A\}$. Let $A \subseteq X$ and $x \in X$ either $x \in A or x \notin A$

-if $x \notin A$ then A not s-coc- neighborhood of x hence $A \notin Ns-coc(x)$

-if $x \in A$ .Since $\{x\} \subseteq X$, and $\{x\}$ open set in X . Then $\{x\}$ s-coc-open set $x \in \{x\} \subseteq A$ $then A$ s-coc- neighborhood of x then $A \in N_{s-coc}(x)$ hence $N_{s-coc}(x) = \{A : x \in A\}$

Remark (1.8)

Every neighborhood of x is s-coc- neighborhood of x . But the converse not true for example

Example (1.3)

Let $x = \{1, 2, 3, 4\} and \tau = \{\emptyset, X, \{4\}\}$, $A = \{1, 2\}$ not neighborhood of x but s-coc- neighborhood of x

Definition (1.10)

Let X be a topological space and $x \in X, A \subseteq X$ .The point x is called s-coc-limit point of A if every s-coc-open set containing x contains a point of A distinct from x. we call the set of all s-coc-limit point of A the s-coc-derived set of A and denoted by A'^{s-coc}. Therefore $x \in A'^{s-coc}$ if and only if for every s-coc-open set $V in X$ such that $x \in V$ such that $(V \cap A) — \{x\} \neq \emptyset$

Definition (1.11)[7]

Let X be a topological space and $x \in X, A \subseteq X$ .The point x is called coc-limit point of A if every coc-open set containing x contains a point of A distinct from x. we call the set of all coc-limit point of A the coc-derived set of A and denoted by A'^{coc}.

Therefore $x \in A'^{coc}$ if and only if for every coc-open set $V in X$ such that $x \in V$ such that $(V \cap A) — \{x\} \neq \emptyset$

Proposition (1.16)

Let X be a topological space and $A \subseteq B \subseteq X$ then

1) $\overline{A}^{s-coc} = A \cup \acute{A}^{s-coc}$

2) A s-coc-closed set if and only if $A'^{s-coc} \subseteq A$

3) $\acute{A}^{s-coc} \subseteq \acute{B}^{s-coc}$

4) $\acute{A}^{s-coc} \subseteq \acute{A}^{coc}$

Proof:

1) $Since$ $A \subseteq \overline{A}^{s-coc}$ ---- (*) , let $x \in \acute{A}^{s-coc}$, then for each s-coc-open set U containing x such that $(V \cap A) - \{x\} \neq \Phi$ then $U \cap A \neq \emptyset$ for all U s-coc-open set, x ∈ U then U∩ $A \neq \emptyset$ .Then $x \in \overline{A}^{s-coc}$ hence $\acute{A}^{s-coc} \subseteq \overline{A}^{s-coc}$ …….(**)from (*)and(**) we have $A \cup A'^{s-coc} \subseteq \overline{A}^{s-coc}$

Conversely:

Let $x \in \overline{A}^{s-coc}$ Then either $x \in A or x \notin A$, if $x \in A$ $then$ $x \in A \cup \grave{A}^{s-coc}$ complete if $x \notin A$, since $x \in \overline{A}^{s-coc}$ then for all U s-coc-open set contains x such that $U \cap A \neq \emptyset$ since $x \notin A$ $then$ $(U \cap A) - \{x\} \neq \emptyset$. Then $x \in A'^{s-coc}$ then $x \in A \cup A'^{s-coc}$

Hence $\overline{A}^{s-coc} \subseteq A \cup A'^{s-coc}$ $then$ $\overline{A}^{s-coc} = A \cup \acute{A}^{s-coc}$.

2) Let A s-coc-closed set to prove $A'^{s-coc} \subseteq A$, $A then x \in A^c$, let $A then A^c$ since A s-coc-closed set, then A^c s-coc set and $A \cap A^c = \emptyset. Since x \notin A then (A \cap A^c) - \{x\} \neq \emptyset$ $then$ $x \notin A'^{s-coc}$ $then A'^{s-coc} \subseteq A$, let $A'^{s-coc} \subseteq A$, to prove A s-coc-closed set.Since $\overline{A}^{s-coc} = A \cup A'^{s-coc}$ then $\overline{A}^{s-coc} = A$ then A then A s-coc-closed set

3)Let $x \in A'^{s-coc}$ .Then for all U s-coc-open set contain x such that $(U \cap A) - \{x\} \neq \emptyset$ .since $A \subseteq B$ $then$ $(U \cap B) - \{x\} \neq \emptyset$. Then $x \in \acute{B}^{s-coc}$ hence $\acute{A}^{s-coc} \subseteq \acute{B}^{s-coc}$

4)By Definition (1.10) and Definition (1.11)

Definition (1.12)

Let (X, τ) be a topological space and let A be any subset of X, let $x \in X$ is called s-coc-boundary point of A if and only if each s-coc-open set U_x of x we have $U_x \cap A \neq \emptyset$ and $U_x \cap A^c \neq \emptyset$

The set of all s-coc-boundary point of A is denoted by $b_{s-coc}(A)$

Proposition (1.17)

Let X be a space and $A \subseteq X$ then

1) $b_{s-coc}(A) = \overline{A}^{s-coc} \cap \overline{A^c}^{s-coc}$

2) $A^{\circ s-coc} = A - b_{s-coc}(A)$

3) $\overline{A}^{s-coc} = A \cup b_{s-coc}(A)$

Proof:

1) Let $x \in b_{s-coc}(A)$ then for all U s-coc-open set contain $x such\ that\ U \cap A \neq \emptyset\ and U \cap A^c \neq \emptyset$. Then by proposition (1.5) we have $x \in \bar{A}^{s-coc}\ and x \in \overline{A^c}^{s-coc}$. Hence $x \in \overline{A}^{s-coc} \cap \overline{A^c}^{s-coc}$. Then $b_{s-coc}(A) \subseteq \overline{A}^{s-coc} \cap \overline{A^c}^{s-coc}$

Conversely:

Let $x \in \overline{A}^{s-coc} \cap \overline{A^c}^{s-coc}$. $To\ proof\ x \in b_{s-coc}(A)$. Then $x \in \overline{A}^{s-coc}\ and x \in \overline{A^c}^{s-coc}$. Then by proposition (1.6) we have U s-coc-open set containing $x and U \cap A \neq \emptyset, U \cap A^c \neq \emptyset$. Then $x \in b_{s-coc}(A)$ by definition (1.12). Therefore $b_{s-coc}(A) \subseteq \overline{A}^{s-coc} \cap \overline{A^c}^{s-coc}$. Thus $b_{s-coc}(A) = \overline{A}^{s-coc} \cap \overline{A^c}^{s-coc}$

2) Let $x \in A^{\circ s-coc}\ then x \in A$ ($since\ A^{\circ s-coc} \subseteq A$). To prove $x \notin b_{s-coc}(A)$, let $x \in b_{s-coc}(A)$, then for all U s-coc-open set contain $x such\ that\ U \cap A^c \neq \emptyset\ and U \cap A \neq \emptyset$. Since $x \in A^{\circ s-coc}$ .Then $there\ exist\ V$ s-coc-open set such that $x \in V \subseteq A$ by proposition (1.10)since $A \cap A^c = \emptyset then V \cap A^c = \emptyset$ contradiction Then $x \notin b_{s-coc}(A)$. Hence $A^{\circ s-coc} \subseteq A - b_{s-coc}(A)$.

Conversely:

Let $x \in A - b_{s-coc}(A)$, to prove $x \in A^{\circ s-coc}$. Since $x \notin b_{s-coc}(A)$ then V s-coc-open set contain x such that $V \cap A \neq \emptyset or V \cap A^c \neq \emptyset$, since $x \in V and x \in A then A \cap V \neq \emptyset then V \cap A^c = \emptyset$ then $V \subseteq A$ then $x \in V \subseteq A$. Then byproposition (1.10) we have $x \in A^{\circ s-coc}$.

3) Let $x \in \overline{A}^{s-coc}$ .To prove $x \in A \cup b_{s-coc}(A)$, let $x \notin A \cup b_{s-coc}(A)$ $then\ x \notin A and x \notin b_{s-coc}(A)$ then there exists V s-coc-open set such that $x \in V and\ V \cap A \neq \Phi\ or V \cap A^c \neq \emptyset$, since $x \notin A\ then\ x \in A^c$, since $x \in V then x \in V \cap A^c$ then $V \cap A^c \neq \emptyset$ hence $V \cap A = \Phi then\ x \notin \overline{A}^{s-coc}$ this contradiction, then $x \in A \cup b_{s-coc}(A)$

Conversely:

Let $x \in A \cup b_{s-coc}(A)$. Then either $x \in A\ then\ x \in \overline{A}^{s-coc}$ or $x \in b_{s-coc}(A)$, then $x \in \overline{A}^{s-coc} \cap \overline{A^c}^{s-coc}$. $then\ x \in \overline{A}^{s-coc}$. Then $A \cup b_{s-coc}(A) \subseteq \overline{A}^{s-coc}$ .Then $\overline{A}^{s-coc} = A \cup b_{s-coc}(A)$

Remark (1.9)

Let X be a topological space and $A \subseteq X$ then $b_{s-coc}(A) = b_{s-coc}(A^c)$

Proof

Since

$b_{s-coc}(A) = \overline{A}^{s-coc} \cap \overline{A^c}^{s-coc} = \overline{(A^c)^c}^{s-coc} \cap \overline{A^c}^{s-coc} = b_{s-coc}(A^c)$

Proposition (1.18)

Let X be a topological space and $A \subseteq X$ then:

i. $\overline{A}^{s-coc} = A^{\circ s-coc} \cup b_{s-coc}(A)$
ii. A s-coc-open set if and only if $b_{s-coc}(A) \subseteq A^c$
iii. A s-coc-closed set if and only if $b_{s-coc}(A) \subseteq A$

Proof

i. Since $A^{\circ s-coc} \subseteq \overline{A^c}^{s-coc}$, $b_{s-coc}(A) = \overline{A}^{s-coc} \cap \overline{A^c}^{s-coc} \subseteq \overline{A}^{s-coc}$ .Then $A^{\circ s-coc} \cup b_{s-coc}(A) \subseteq \overline{A}^{s-coc}$

Conversely:

$x \in \bar{A}^{s-coc}\ and x \notin b_{s-coc}(A)$then there exist U s-coc-open set contain $x and$ $U \cap A = \emptyset\ \ or\ \ U \cap A^c = \emptyset$. if $U \cap A = \emptyset$ then $x \notin \overline{A}^{s-coc}$ contradiction, if $U \cap A^c = \emptyset$ then $U \subseteq A$, then there exist U s-coc-open such that $x \in U and U \subseteq A\ then\ x \in A^{\circ s-coc}$

ii. Let A s-coc-open set.Then A^c s-coc-closed set $then\ A^\circ \subseteq \overline{A^c}^{s-coc}$. Since $b_{s-coc}(A) = \overline{A}^{s-coc} \cap \overline{A^c}^{s-coc} \subseteq \overline{A^c}^{s-coc}$. Hence $b_{s-coc}(A) \subseteq \overline{A^c}^{s-coc}$

Conversely:

Let $b_{s-coc}(A) \subseteq A^c$. Since $b_{s-coc}(A) = \overline{A}^{s-coc} \cap \overline{A^c}^{s-coc}$. Then $\overline{A}^{s-coc} \cap \overline{A^c}^{s-coc} \cap A = \emptyset$. Since $A \subseteq \overline{A}^{s-coc}\ then\ \overline{A^c}^{s-coc} \cap A = \emptyset$. Then $\overline{A^c}^{s-coc} \subseteq A^c$. Since $A^c \subseteq \overline{A^c}^{s-coc}\ hence A^c = \overline{A^c}^{s-coc}$. Then A^c s-coc-closed $then\ A$ s-coc-open

iii. Let A s-coc-closed set .Then $A = \overline{A}^{s-coc}\ then\ b_{s-coc}(A) = \overline{A}^{s-coc} \cap \overline{A^c}^{s-coc} = A \cap \overline{A^c}^{s-coc} \subseteq A$

Conversely

Let $b_{s-coc}(A) \subseteq A$.Then $\overline{A}^{s-coc} \cap \overline{A^c}^{s-coc} \cap A^c = \emptyset then\ \overline{A}^{s-coc} \cap A^c = \emptyset\ \ then\ \ \overline{A}^{s-coc} \subseteq A hence A = \overline{A}^{s-coc}$, Then A s-coc-closed set.

Proposition (1.19)

If X aspace and τ discrete topology on X then $b_{s-coc}(A) = \emptyset for all A \subseteq X$

Proof

Since X discrete then A s-coc-open set, Then $b_{s-coc}(A) \subseteq A^c$ and A s-coc-closed set, Then $b_{s-coc}(A) \subseteq A$ by proposition (1.18)(ii) . Then $b_{s-coc}(A) \subseteq A \cap A^c = \emptyset$.Thus $b_{s-coc}(A) = \emptyset$ by proposition (1.18)(iii)

Definition (1.13) [6]

A subset A is said to be ω-open set if for each$x \in A$, there exists open set U such that $x \in U\ and\ A - U$ countable

Definition (1.14)

A subset A is said to be ω s-coc-open set if for each$x \in A$, there exists open s-coc-open set U such that $x \in U\ and\ A - U$ countable. The complement of ω coc-open set

isω coc-closed set.

Proposition (1.20)

A subset A of space X is ωs-coc-open set if and only if for each $x \in A$ there exists open s-coc-open set U containing x and countable subset B such that $U - B \subseteq A$

Proof

Let A be ω s-coc-open and $x \in A$, there exist s-coc-open subset U containing x such that $U - A$ countable

Let $B = U - B = U \cap (X - A)$ then $U - B \subseteq A$

Conversely:

Let $x \in A$ and there exists s-coc-open subset U containing x and countable subset B such that $U - B \subseteq A$. Then $U - A$countable set. Then A is ωs-coc-open set

Theorem (1.1)

Let X be a space and $C \subseteq X$. If C is ω s-coc-closed set.Then $C \subseteq K \cap B$ for some s-coc-closed subset K and countable subset B

Proof

Since C is ωs-coc-closed set *then* $X - C\omega$s-coc-open. Then by proposition (1.20) *then* $\forall x \in X - C \; \exists \, U$ s-coc-open and B countable set such that $U - B \subseteq X - C$. Then $C \subseteq X - (U - B)$

$$= X \; = X \cap \left(U \cap (X - B)\right)^c = X \cap (U^c \cup B)$$

$$= (X - U) \cup B$$

Let $K = X - U$ *then* $C \subseteq K \cap B$

Proposition (1.21)

The intersection of two ω s-coc-open set is ω s-coc-open set

Proof

Let A *and* B ω s-coc-open sets and $x \in A \cap B$ *then* $x \in A$ *and* $x \in B$. Since A ωs-coc *then* $\forall x \in A \; \exists \, U$ s-coc-open such that $U - A$countable.Since B ωs-coc then$\forall x \in B \; \exists \, V$ s-coc-open such that $V - B$countable .To prove $(U \cap V) - (A - B)$ countable

$(U \cap V)$s-coc-open set by Remark (1.1.4)(2)and $x \in U \cap V$

$$(U \cap V) - (A - B) = (U \cap V) \cap [(X - A) \cup (X - B)]$$

$$= [(U \cap V) \cap (X - A)] \cap [(U \cap V) \cap (X - B)]$$

$$\subseteq (U - A) \cup (V - B)$$

Since$U - A$ *and* $V - B$countable.Then $(U - A) \cup (V - B)$countable.Then $(U \cap V) - (A - B)$ countable

Proposition (1.22)

The union of any family of ω s-coc-open set is w s-coc-open set

Proof

Let $x \in \cup A_\propto and A_\propto \omega$ s-coc-open set .Then $x \in A_\propto$for some$\propto \in \Lambda$.Then there exists subset U s-coc-open and B countable such that $U - B \subseteq A_\propto$. Then $U - B \subseteq A_\propto \subseteq \cup A_\propto$. Then $\cup A_\propto$ ωs-coc-open set

Proposition (1.23)

For space X then

i. *Every* ωopen is ωs-coc-open set

ii. Every s-coc-open is ωs-coc-open set

Proof

i. Let $A\omega$-open set.Then for all $x \in A$ there exist U open set such that $x \in U$ and $U - A$countable. Since every open is s-coc-open set by Remark (1.3)(i). Then U s-coc-open set. Thus for all $x \in A$ there is U s-coc-open such that $U - A$countable

ii. Let A s-coc-open set .Then for all $x \in A$ there exist $V = A$ s-coc-open set containing x such that $V - A = A - A = \emptyset$ countable. Therefore $A\omega$ s-coc-open set.

Definition (1.15)[7]

A subset A is said to be ω coc-open set if for each $x \in A$, there exists open coc-open set U such that $x \in U$ *and* $A - U$ countable.The complement of ω coc-open set is ω coc-closed set.

Proposition (1.24)

Every ω coc-open set is ω s-coc-open set.

Proof

Let A is ω coc-open set and $x \in A$ then there exists U coc-open set such that $x \in U$ and U—A countable. Since every coc-open set is s-coc-open set by Remark (1.3) (i) then Uiss-coc-open then A is ω s-coc-open set**.**

Proposition (1.25)

The intersection of ω s-coc-open set ω coc-open set is ω s-coc-open

Proof

Le tA is ω s-coc-open set and B is ω coc-open set, then B ω s-coc-open set by Proposition (1.24). Then A $\cap$B is ω s-coc-open set by Proposition (1.21).

Proposition (1.26)

Every s-open set is ω s-coc-open set

Proof

Let A is s-open set then A is s-coc-open set. Then A is ω s-coc-open set.

$$\begin{tikzcd}
\omega\text{coc-open} \arrow[r] & \omega \text{ s-coc-open} & \text{s-open} \arrow[l] \\
\omega \text{ open} \arrow[u] \arrow[r] & \text{s-coc-open} \arrow[u] &
\end{tikzcd}$$

2. On s-coc-open and super s-coc-open function

We introduce and study s-coc-open and s-coc-closed function also some properties about them

Definition (2.1) [2]

Let $f\colon X \longrightarrow Y$ be a function of space X into space Y then:

i- $f$is called open function if $f(A)$ is open set in Y for every open set A *in* X.

ii- f is called closed function if $f(A)$ is closed set in Y for every closed set A *in* X.

Theorem (2.1) [4]

Let $f\colon X \longrightarrow Y$ be a function of space X into space Y then the following statements are equivalent.

i- f open function.

ii- $f(A^\circ) \subseteq \left(f(A)\right)^\circ$for every subset A of X.

iii- $\left(f^{-1}(A)\right)^\circ \subseteq f^{-1}(A^\circ)$for every subset A of X.

iv- $f^{-1}(\bar{A}) \subseteq \overline{f^{-1}(A)}$for every subset A of X.

Definition (2.2) [7]
Let $f: X \longrightarrow Y$ be a function of space X into space Y then:
i- f is called coc-closed function if $f(A)$ is coc-closed set in Y for every closed set A in X.
ii- f is called coc-open function if $f(A)$ is coc-open set in Y for every open set A in X.

Definition (2.3)
Let $f: X \longrightarrow Y$ be a function of space X into space Y then:
1) f is called s-open function if $f(A)$ is s-open set in Y for every open set A in X [1].
2) f is called s-closed function if $f(A)$ is s-closed set in Y for every closed set A in X.

Definition (2.4)
Let $f: X \longrightarrow Y$ be a function of space X into space Y then:
1) f is called s-coc-closed function if $f(A)$ is s-coc-closed set in Y for every closed set A in X.
2) f is called s-coc-open function if $f(A)$ is s-coc-open set in Y for every open set A in X.

Proposition (2.1)
1. Every s-closed function is s-coc-closed function
2. Every s-open function is s-coc-open function.
3. Every coc-open function is s-coc-open.

Proof
1- Let $f: X \longrightarrow Y$ s-closed function, let$B$ closed set in X.Then $f(B)$ is s-closed set in Y *then* $(f(B))^c$ s-open set then $(f(B))^c$ s-coc-open set.Then $f(B)$ s-coc-closed set in Y.Therefore f s-coc-closed function.
2-Let $f: X \longrightarrow Y$ s-open function.Let A open set in X then $f(A)$ is s-open set in Y. Since every s-open set is s-coc-open then $f(A)$ is s-coc-open set.Hence f s-coc-open function.

3- Let $f: X \longrightarrow Y$coc-openfunction, let A open set in X *then* $f(A)$ is coc-open set in Y.*then* $f(A)$is coc-open set in Y. Then f is s-coc-open function .The inverse of 1. and 3.is not hold for examples

Examples (2.1)
1. Let $X = \{1,2,3\}, \tau = \{\emptyset, X, \{1,2\}\}$ *topology on* X , $Y = \{a, b\}$, $\tau = \{\emptyset, Y, \{b\}\}$ *topology on* Y , $f(1) = f(2) = a$, $f(3) = b$, since $\{1,2\}$ open set in X and$f(\{1,2\}) = \{a\}$ not open in$Y$, *since* $\{a\}^\circ = \emptyset$ *then* $\{a\} \not\subset \overline{\{a\}}^\circ$. But $f(\{1,2\})$ s-coc-open set Then$f$s-coc-open but not s-open.
3. Let $X = \{1,2,3,\dots\}, \tau = \{\emptyset, X, \{2\}\}$ and $Y = \{y_1, y_2, y_3, \dots\}, \tau^* = \{\emptyset, Y, \{y_1\}\}$, let $f: X \longrightarrow Y$ defined by $f(1) = y_1, (2) = y_2, f(\{1,2\}^c) = \{y_1, y_2\}^c$. Since$\{2\}$ open in X and $f(2) = f(y_2)$ not coc-open in Y then f not coc-open function . But τ^{sk} is discrete topology on X and τ^{*sk} is discrete topology on Y then f s-coc-open function.

Proposition (2.2)
A function $f: X \longrightarrow Y$ s-coc-closed if and only if $\overline{f(A)}^{s-coc} \subseteq f(\bar{A})$ for all$A \subseteq X$.
Proof
Suppose $f: X \longrightarrow Y$ s-coc-closed function, let $A \subseteq X$. Since $\bar{A}$ closed set in X then $f(\bar{A})$ s-coc-closed in Y. Since $A \subseteq \bar{A}$then $(A) \subseteq f(\bar{A})$. Hence $\overline{f(A)}^{s-coc} \subseteq \overline{f(\bar{A})}^{s-coc}$ .Since $f(\bar{A}) \subseteq \overline{f(\bar{A})}^{s-coc}$ .Then $\overline{f(A)}^{s-coc} \subseteq f(\bar{A})$.
Conversely
Let B closed set in Xthen $= \bar{B}$. Then $\overline{f(B)}^{s-coc} \subseteq f(\bar{B}) = f(B)$. Since $f(B) \subseteq \overline{f(B)}^{s-coc}$.Then $f(B) = \overline{f(B)}^{s-coc}$.Hence $f: X \longrightarrow Y$ s-coc-closed function.

Proposition (2.3)
A function $f: X \longrightarrow Y$ s-coc-open if and only if $f(A^\circ) \subseteq (f(A))^{\circ\, s-coc}$ for all $A \subseteq X$.
Proof
Suppose $f: X \longrightarrow Y$ s-coc-open function and A $\subseteq X$, since A° open in X .then $f(A^\circ)$ s-coc-open in Y Then $(f(A^\circ))^{\circ s-coc} \subseteq (f(A))^{\circ\, s-coc}$.Therefore $f(A^\circ) = (f(A^\circ))^{\circ s-coc} \subseteq (f(A))^{\circ\, s-coc}$.Then $f(A^\circ) \subseteq (f(A))^{\circ\, s-coc}$ for all $A \subseteq X$.
Conversely
Let A open in X. Then $A = A^\circ$, Since$f(A^\circ) \subseteq (f(A))^{\circ\, s-coc}$ .Then $f(A) \subseteq (f(A))^{\circ\, s-coc}$.Then $f(A) = (f(A))^{\circ\, s-coc}$.Therefore $f: X \longrightarrow Y$ s-coc-open function.

Definition (2.5)
Let $f: X \longrightarrow Y$ be a function of a space X in to a space Y. $f$is called s-coc-continuous function if $f^{-1}(A)$ is open set in X for every open set in Y.

Theorem (2.2)
For bijective function$f: (X, \tau) \longrightarrow (Y, \acute{\tau})$ the following statements are equivalent.
1) $f^{-1}$is s-coc-continuous.
2) $f$is s-coc-open.
3) $f$is s-coc-closed.

Proof
$1 \longrightarrow 2$
Let A open set in X and $f^{-1}: Y \longrightarrow X$ s-coc-continuous, then $(f^{-1})^{-1}(A)$ s-coc-open in X, since f bijective then $f(A) = (f^{-1})^{-1}(A)$ .Then $f: X \longrightarrow Y$ s-coc-open.

$2 \longrightarrow 3$
Let $f: X \longrightarrow Y$ s-coc-open function and B closed set in X . Then B^c open set in , hence $f(B^c)$ s-coc-open in Y .Since $f(B^c) = (f(B))^c$. Then $(f(B))^c$ s-coc-open.Then $f(B)$ s-coc-closed.Therefore f s-coc-closed function.
$3 \longrightarrow 1$
Let f s-coc-closed function.To prove $f^{-1}: Y \longrightarrow X$ s-coc-continuous.Let F closed set in X.Since f s-coc-closed.then$f(F)$ s-coc-closed.Since f bijective then $f(F) = (f^{-1})^{-1}(F)$. Then $f^{-1}: Y \longrightarrow X$ s-coc-continuous

Not that: -
The composition of two s-coc-open functions is not s-coc-open function for example:

Example (2.2)

Let $f: \mathcal{R} \longrightarrow \mathcal{Z}_e$ function defined by $f(x) = \begin{cases} 0 & if\ x = 0 \\ \mathcal{Z}_e{}^{+} & if\ x \in \mathcal{R}^{+} \\ \mathcal{Z}_e{}^{-} & if\ x \in \mathcal{R}^{-} \end{cases}$ and $g: \mathcal{Z}_e \longrightarrow \mathcal{Z}_\circ$ defined by $g(x) = \begin{cases} \mathcal{Z}_\circ{}^{+} & if\ x = 0 \\ \mathcal{Z}_\circ{}^{-} & if\ x \neq 0 \end{cases}$, $\tau = \{\emptyset, \mathcal{R}, \{0\}\}$ $topology\ on\ \mathcal{R}\ and\ \acute{\tau} = \{\emptyset, \mathcal{Z}_e, \{-4, -2\}\}$ $topology\ on\ \mathcal{Z}_e$, $\tau'' = \{\emptyset, \mathcal{Z}_\circ, \mathcal{Z}_\circ{}^{-}\}$ $topology\ on\ \mathcal{Z}_\circ$. Then f s-coc-open function and g s-coc-open function.But $(g \circ f)(\{0\}) = g(f(0)) = \mathcal{Z}_\circ{}^{+}$ .Since $1 \in \mathcal{Z}_\circ{}^{+}$ and the s-open set contain $1\ is\ \mathcal{Z}_\circ$and K any compact set . But $1 \in \mathcal{Z}_\circ{}^{+} - K \nsubseteq \mathcal{Z}_\circ{}^{+}$.Then $\mathcal{Z}_\circ{}^{+}$ not s-coc-open set.Then $(g \circ f)(\{0\})$ is not s-coc-open set. Then $(g \circ f)$ is not s-coc-open function.

Proposition (2.4)

If $f: X \longrightarrow Y$ open function and $g: Y \longrightarrow W$ s-coc-open function then $g \circ f$ s-coc-open function.

Proof

Let A open set in X . Since f open function. Then $f(A)$ open set in Y .Since g open function. Then $g(f(A))$open set in W .Then $g \circ f: X \longrightarrow W$ s-coc-open function.

Definition (2.6)

Let $f: X \longrightarrow Y$ be a function of space X into space Ythen:

i- f is called s-coć-closed function if $f(A)$ is s-coc-closed set in Y for all s-coc-closed A in X.

ii- f is called s-coć-open function if $f(A)$ is s-coc-open set in Y for all s-coc-open A in X.

Definition (2.7)[7]

Let $f: X \longrightarrow Y$ be a function of space X into space Ythen:

1. f is called coć-closed function if $f(A)$ is coc-closed set in Y for all coc-closed A in X.
2. f is called coć-open function if $f(A)$ is coc-open set in Y for all coc-open A in X.

Proposition (2.5)

A function $f: X \longrightarrow Y$ s-coć-open if and only if $f(A^{\circ s-coc}) \subseteq (f(A))^{\circ s-coc}$ for all $A \subseteq X$.

Proof

Let $f: X \longrightarrow Y$ s-coć-open function, let $A \subseteq X$.Since $A^{\circ s-coc}$ s-coc-open in X .Then $f(A^{\circ s-coc})$ s-coc-open in Y. Then$f(A^{\circ s-coc}) = (f(A^{\circ s-coc}))^{\circ s-coc}$ .Since$A^{\circ s-coc} \subseteq A$then $f(A^{\circ s-coc}) \subseteq f(A)$Then$f(A^{\circ s-coc}) = (f(A^{\circ s-coc}))^{\circ s-coc} \subseteq (f(A))^{\circ s-coc}$.Then$f(A^{\circ s-coc}) \subseteq (f(A))^{\circ s-coc}$ $\forall A \subseteq X$.

Conversely

Let A s-coc-open in X .Then $A = A^{\circ s-coc}$. Since $f(A^{\circ s-coc}) \subseteq (f(A))^{\circ s-coc}$.Then $f(A) \subseteq (f(A))^{\circ s-coc}$ .Thus $f(A) = (f(A))^{\circ s-coc}$.Then $f: X \longrightarrow Y$ s-coć-open function.

Proposition (2.6)

If $f: X \longrightarrow Y$ s-coć-open then f is s-coc-open

Proof

Let A open set in X. Then A s-coc-open. Since f is s-coć-open then then $f(A)$ is s-coc-open set in Y then f is s-coc-openfunction

Not that

If f s-coć-open function then it need not to be coć-open for example

Example (2.3)

Let$f: X \longrightarrow Y$, X={1, 2, 3, ...}and $\tau = \{\emptyset, X, \{1\}\}$, Y= $\{y_1, y_2, y_3, \ldots\}and\tau = \{\emptyset, Y, \{y_2, y_3\}\}$, $f(i) = y_i$ when $i = 1, 2, 3, \ldots, since f(1) = y_1$not coc´-open set in Y then $f$is not coc´-open function. But $f$is s-coc´-open function

Proposition (2.7)

If $f: X \longrightarrow Y$ and $g: Y \longrightarrow Z$ s-coć-open functions then $g \circ f$ s-coć-open function.

Proof

Clear

Proposition (2.8)

A function $f: X \longrightarrow Y$is s-coć-closed if and only if $\overline{f(A)}^{s-coc} \subseteq f(\overline{A}^{s-coc})$ for all $A \subseteq X$.

Proof

Suppose $f: X \longrightarrow Y$is s-coć-closed function and $A \subseteq X$ .Since $\overline{A}^{s-coc}$ s-coc-closed set in X.Then $f(\overline{A}^{s-coc})$ s-coc-closed set in Y ……..($*$)

Since $A \subseteq \overline{A}^{s-coc}$ $then\ f(A) \subseteq f(\overline{A}^{s-coc})$.Hence $\overline{f(A)}^{s-coc} \subseteq \overline{f(\overline{A}^{s-coc})}^{s-coc}$ .Since $f\left(\overline{A}^{s-coc}\right) = \overline{f(\overline{A}^{s-coc})}^{s-coc}$ by $(*)$. Then $\overline{f(A)}^{s-coc} \subseteq f(\overline{A}^{s-coc})$

Conversely:

Let F s-coc-closed set in X then $F = \overline{F}^{s-coc}$.Then $\overline{F}^{s-coc} \subseteq f\left(\overline{F}^{s-coc}\right) = f(F)$.Since $f(F) \subseteq \overline{F}^{s-coc}$.Then $f(F) = \overline{F}^{s-coc}$. Then $f$is s-coć-closed function.

Definition (2.8)

Let $f: X \longrightarrow Y$ be a function of a space X in to a space Y then f is called s-coc-irresolute ($s\text{–}coć\text{–}continuous$) function if $f^{-1}(A)$ s-coc-open set in X for every s-coc-open set $in Y$.

Theorem (2.3)

For a bijective mapping $f: (X, \tau) \longrightarrow (Y, \acute{\tau})$ then the following statements are equivalent

1) $f^{-1}$is s-coć-continuous
2) f is s-coć-open function
3) f is s-coć-closed function

Proof

$1 \longrightarrow 2$

Let A s-coc-open set in X and $f^{-1}: Y \longrightarrow X$ s-coć-continuous then $(f^{-1})^{-1}(A)$ is s-coć-open in Y .since

$f$bijective, then $f(A) = (f^{-1})^{-1}(A)$ .Then $f(A)$ s-coć-open in Y .Thus $f: X \longrightarrow Y$ is s-coć-open in Y

$2 \longrightarrow 3$

Let $f: X \longrightarrow Y$ is s-coć-open and F s-coc-closed set in X .Then F^cs-coc-open set in X Since f is s-coć-open function .Then $f(F^c)$ s-coc-open in Y , Since $f(F^c) = (f(F))^c$.Then $(f(F))^c$ s-coc-open .Then $f(F)$ s-coc-closed .Thus f is s-coć-closed function.

$3 \longrightarrow 1$

Let $f: X \longrightarrow Y$ is s-coć-closed function and B s-coc-closed set in XThen $f(B)$ s-coc-closed set in Y, since f bijective, then $f(B) = (f^{-1})^{-1}(B)$ Then $f^{-1}: Y \longrightarrow X$ s-coć-continuous.

Definition (2.9)

Let X *and* Y are spaces. Then a function $f: X \longrightarrow Y$ is called s-coc-homeomorphism if

1. f bijective
2. f s-coc-continuous
3. f s-coc-closed (s-coc-open)

It is clear that every homeomorphism is s-coc-homeomorphism

Definition (2.10)

Let X *and* Y are spaces. Then a function $f: X \longrightarrow Y$ is called s-coć-homeomorphism if :

1. f bijective
2. f s-coć-continuous
3. f s-coć-closed (s-coc-open)

It is clear that every homeomorphism is s-coc-homeomorphism

Proposition (2.11)

Let $f: (X, \tau) \longrightarrow (Y, \acute{\tau})$ bijective function then the following statements are equivalent

i- f is s-coć-homeomorphism

ii- f s-coć-continuous and closed

Proof

$i \longrightarrow ii$by definition of s-coć-homeomorphism

$i \longrightarrow ii$ clear.

Proposition (2.12)

Let $f: X \longrightarrow Y$ is s-coć-homeomorphism then $f(B^\circ) \subseteq (f(B))^{\circ s-coc}$ $\forall B \subseteq X$

Proof

Since f is s-coc-homeomorphism then f is s-coc-homeomorphism. Then f is s-coc-open function .Then $f(B^\circ) \subseteq (f(B))^{\circ s-coc}$ by proposition (2.3).

Theorem (2.4)

If $f: (X, \tau) \longrightarrow (Y, \acute{\tau})$ s-coc-open function then for all $x \in X$ and all nbd U *of* x there exists V s-coc-open set in Y containing $f(x)$ such that $V \subseteq f(U)$

Proof

Let U be nbd of x in X then there exists an open set B such that $x \in B \subseteq U$ hence $f(x) \in f(B) \subseteq f(U)$, Since $f(B)$ s-coc-open set in Y .Let $V = f(B)$ then $V \subseteq f(U)$

Theorem (2.5)

If $f: (X, \tau) \longrightarrow (Y, \acute{\tau})$ s-coć-open function then for all $x \in X$ and all s-coc-nbd A *of* x there exists B s-coc-open set in Y containing $f(x)$ such that $B \subseteq f(A)$.

Proof

Let $x \in X$ *and* A s-coc-nbd of x.Then there exists F s-coc-open set such that $x \in F \subseteq A$. Then $f(x) \in f(F) \subseteq f(A)$, Since f s-coć-open then $f(F)$ s-coc-open set in Y , Let $B = f(F)$ then $f(x) \subseteq B \subseteq f(A)$

Definition (2.11)

A function $f: (X, \tau) \longrightarrow (Y, \acute{\tau})$ is called

i- super s-coc-open if $f(U)$ is open in Y for each U s-coc-open in X.

ii- super s-coc-closed if $f(U)$ is closed in Y for each U s-coc-closed in X.

Proposition (2.13)

Every super s-coc-open function is s-coc-open

Proof

Let $f: (X, \tau) \longrightarrow (Y, \acute{\tau})$ super s-coc-open and$A$ open set in X.Then A s-coc-open. Since f super s-coc-open then $f(A)$ s-coc-open in Y then s-coc-open function.But the converse is not true for the following example

Example (2.4)

Let $X = \{1,2,3\}, \tau = \{\emptyset, X\} and\ Y = \{a, b\}, \acute{\tau} = \{\emptyset, Y, \{a\}\}$, let $f: (X, \tau) \longrightarrow (Y, \acute{\tau})$ defined by $f(X) = \begin{cases} a & if\ x \in \{1,2\} \\ b & if\ x \in \{3\} \end{cases}$, since$\{3\}$ s-coc-open in X and $f(\{3\}) = \{b\}$ not open in Y. Then$f$ is not super s-coc-open function But f s-coc-open

Proposition (2.14)

1. Every super s-coc-open function is s-coć-open
2. Every super s-coc-open function is s-open

Proof

1. Clear
2. Let A open in X. then A s-coc-open. Since f super s-coc-open then $f(A)$ is open. Then $f(A)$ s-open

But the converse is not true for the following examples

Example (2.5)

1. Let $X = \{1,2,3\}, \tau = \{\emptyset, X\} and\ Y = \{a, b\}, \acute{\tau} = \{\emptyset, Y, \{a\}\}$, $f(X) = \begin{cases} a & if\ x \in \{1,2\} \\ b & if\ x \in \{3\} \end{cases}$. Then f s-coć-open but not super s-coc-open.
2. Let $X = \{1,2,3\}, \tau = \{\emptyset, X\} and\ Y = \{a, b\}, \acute{\tau} = \{\emptyset, Y\}, f(X) = \begin{cases} a & if\ x \in \{1,2\} \\ b & if\ x \in \{3\} \end{cases}$. Then f s-open but not super s-coc-open.

Proposition (2.15)

If f bijective then f is super s-coc-open iff super s-coc-closed

Proof
Let f super s-coc-open function that $f: X \longrightarrow Y$ and let A s-coc-closed set in X then A^c s-coc-open. Then $f(A^c)$ open in Y, Since f bijective then $f(A^c) = \left(f(A)\right)^c$ then $\left(f(A)\right)^c$ open in Y.Then f super s-coc-closed function
Conversely: - by the same way

Proposition (2.16)
If $f: (X, \tau) \longrightarrow (Y, \acute{\tau})$ super s-coc-open function and bijective then**:**
1. $f(A^{\circ s-coc}) = \left(f(A)\right)^{\circ}$ for all A s-coc-open set in X.
2. $f\left(\overline{B}^{s-coc}\right) = \overline{f(B)}$ for all B s-coc-open set in Y.

Proof
1. Let A s-coc-open set in X .Then $A = A^{\circ s-coc}$. Then $f(A) = f(A^{\circ s-coc})$. Since f super s-coc-open, $f(A)$ open set in Y. Then $f(A) = \left(f(A)\right)^{\circ}$. Then $f(A^{\circ s-coc}) = \left(f(A)\right)^{\circ}$
2. Let B s-coc-closed set in X then $B = \overline{B}^{s-coc}$. Then $f(B) = f(\overline{B}^{s-coc})$.Since f super s-coc-open.Then f super s-coc-closedfunction by proposition (2.13). Then $f(B)$ closed set in, then $f(B) = \overline{f(B)}$. Then $f\left(\overline{B}^{s-coc}\right) = \overline{f(B)}$.

Proposition (2.17)
If $f: (X, \tau) \longrightarrow (Y, \acute{\tau})$ super s-coc-open function and B, C have disjoint s-coc-nbds of X .Then $f(B), f(C)$ are disjoint in Y and have disjoint nbds in Y.

Proof
Let F, K are two disjoint s-coc-nbds of B, C then there exists two s-coc-open sets U, Vsuch that $B \subseteq U \subseteq F$ and $C \subseteq V \subseteq K$, Since f super s-coc-open then $f(U), f(V)$ are open sets in Y , $f(B) \subseteq f(U) \subseteq f(F)$ and $f(C) \subseteq f(V) \subseteq f(K)$. Then $f(F), f(K)$ are disjoint nbds of $f(B), f(C)$.Since $F \cap K = \emptyset$ then $f(F \cap K) = \emptyset$ then $f(F) \cap f(K) = \emptyset$then $f(B) \cap f(C) \subseteq f(U) \cap f(V) \subseteq f(F) \cap f(K) = \emptyset$ .Therefore $f(B), f(C)$ are disjoint in Y.

Theorem (2.6)
If $f: (X, \tau) \longrightarrow (Y, \acute{\tau})$ super s-coc-open function then for each $x \in X$ and each s-coc-nbd U of $x$there exists nbd V of $f(x)$ such that $V \subseteq f(U)$.

Proof
Let $x \in X$ and U nbd of $x$then there exists s-coc-open set A such that $x \in A \subseteq U$then $f(x) \in f(A) \subseteq f(U)$ **,** Since f super s-coc-open and A is s-coc-open set in Xthen $f(A)$ open set in Y **,** let $V = f(A)$ thus $f(x) \in V \subseteq f(U)$

Proposition (2.18)
1. If f super s-coc-open function then f coc-open
2. If f super s-coc-open function then f coc´-open

Proof
1. Let $f: X \longrightarrow Y$ super s-coc-open function and A open set in X then $f(A)$ open set in Y $then$ $f(A)$ coc-open set in Y then f coc-open function
2. Let $f: X \longrightarrow Y$ super s-coc-open function and A coc-open set in X then $f(A)$ open set in Y then $f(A)$ coc-open set in Y then f coc´-open function

Not that

The convers is not hold for examples

Example (2.6)
1. Let X= $Z, \tau = \{\emptyset, Z, Z_e\}$ and Y= $\{a, b\}$, $\tau^* = \{\emptyset, Y, \{b\}\}$ and $f: Z \longrightarrow Y$ defined by $f(X) = \begin{cases} a & if\ x \in Z_e \\ b & if\ x \in Z_o \end{cases}$. It is clear that f coc-open function but not super s-coc-open.
2. Let X$=$\{1, 2, 3\}$\tau = \{\emptyset, X, \{a\}\}$ and Y= $\{a, b, \}$, $\tau^* = \{\emptyset, Y\}$ and $f: Z \longrightarrow Y$ defined by $f(1) =$ a , $f(2) = f(3)$ =b. It is clear that f coc´-open function but not super s-coc-open

Proposition (2.19)
If f super s-coc-open function and bijective then f^{-1} is s-coc-continuous

Proof Clear

Proposition (2.20)
The composition of two super s-coc-open function is super s-coc-open

Proof
Let $f: X \longrightarrow Y$ and $g: Y \longrightarrow W$super s-coc-open and A s-coc-open set in X. Then $f(A)$ is open in Y(since f super s-coc-open), Since every open is s-coc-open. Then $g\left(f(A)\right)$ is open in W.Since$(g \circ f)(A) = g\left(f(A)\right)$hen $g\left(f(A)\right)$ open in W then $g \circ f$ is super s-coc-open function.
Not that
if$f$ super s-coc-open and $g \circ f$is super s-coc-open then g not need super s-coc-open function.

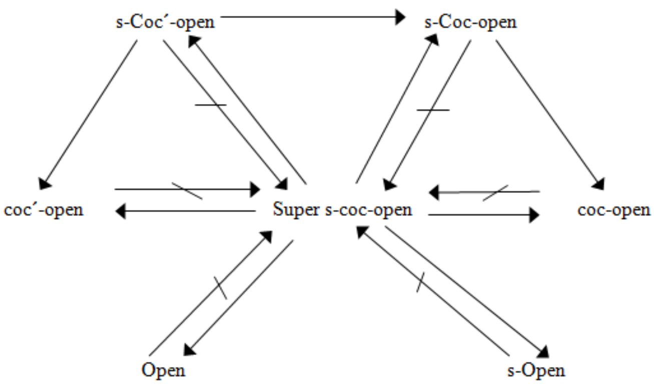
Example (2.7)
Let
$f: (X, \tau) \longrightarrow (Y, \acute{\tau})$ and $g: (Y, \acute{\tau}) \longrightarrow (W, \tau^*)$$and$ $X = Y = W = \{1, 2, 3\}, \tau = \{\emptyset, X, \{1\}\}$ $Topology\ on\ X$, $\acute{\tau} = \{\emptyset, Y, \{1\}, \{2\}, \{1, 2\}\}$ $Topology\ on\ Y$, $\tau^* = \{\emptyset, W, \{3\}\}$$Topology\ on\ W$ such that $f(1) = 1, f(2) = f(3) = 2$ and $g(1) = g(2) = 3, g(3) = 1$ then τ^{sk}on X is discrete Topology then $f$super s-coc-open and $g \circ f$ is super s-coc-open function. But $\{3\}$ s-coc-open in Y and $g(\{3\}) = \{1\}, \{1\}$ not open in .Then g is not super s-coc-open

Not that
If f super s-coc-open and g s-coć-open. then$g \circ f$ is not super s-coc-open for the following example

Example (2.8)
$Let\ f:(X,\tau) \longrightarrow (Y,\acute{\tau})\ and\ \ g:(Y,\acute{\tau}) \longrightarrow (W,\tau^*)$, let X= $\{a_1,a_2,a_3\}$, τ $indiscret\ topology\ on\ X, Y = \{1,2,3\}, \acute{\tau}\ discret\ Topology\ on\ Y\ and\ W = \{b,c\}$, $\tau^*\ indiscret\ topology\ on\ W$ and f defined by $f(x) = \begin{cases} 1 & if\ x = a_1 \\ 2 & if\ x = a_2 \\ 3 & if\ x = a_3 \end{cases}$ and g defined by $g(x) = \begin{cases} b & if\ \ x \in \{1\} \\ c & if\ \ x \in \{2,3\} \end{cases}$, then f super s-coc-open and g is s-coć-open function. But $\{a_1\}$ s-coc-open set in X and $(g \circ f\{a1\} = gf\{a1\} = g\{1\} = \{b\}$ is not open in W. Then $g \circ f$ is not super s-coc-open

Proposition (2.21)
Let $f: X \longrightarrow Y\ and\ g: Y \longrightarrow W$
1. If f super s-coc-open and g s-coc-open then $g \circ f$ s-coć-open.
2. If f s-coc´-continuous, bijective and $g \circ f$ super s-coc-open then g super s-coc-open.

Proof
1. Let$A$ s-coc-open set in X. since f super s-coc-open .Then $f(A)$ open in Y, Since g s-coc-open then $g\big(f(A)\big)$ s-coc-open. Since $(g \circ f)(A) = g\big(f(A)\big)$then $g \circ f$ s-coć-open.
2.Let A s-coc-open set in Y .Then $f^{-1}(A)$ s-coc-open in X (Since $f\ s\text{-}coc'\text{-}continuous$), Since $g \circ f$ super s-coc-open.Then $(g \circ f)(f^{-1}(A))$ open in W Since f bijective.Then $(g \circ f)\big(f^{-1}(A)\big) = g(A)$.Then $g(A)$ super s-coc-open.



3. On s-coc-connected spaces

We recall the concept of s-coc-connected space and give some important generalization on this concept and we prove some results on this concept.

Definition (3.1) [5]
A space X is said to be connected space if X can be expressed as the union of two disjoint-open and non-empty subsets of X other wise, X is connected space.

Definition (3.2) [7]
Let X be a space two subsets A, B of space X are called coc-separated if and only if $\overline{A}^{s-coc} \cap B = A \cap \overline{B}^{s-coc} = \emptyset$.
Not that the family of all s-coc-open subsets of space (X,τ) is denoted by τ^k [10].

Definition (3.3) [7]
Let X be a space and $\emptyset \neq A \subseteq X$.then A is called coc-connected set if and only if is not union of any two coc-separated sets.
Not that (X,τ) is coc-connected if and only if (X,τ^k) is connected.

Remark (3.1) [7]
A set A is called coc-clopen if and only if it is coc-open and coc-closed.

Proposition (3.1) [7]
Let X be space then the following statements are equivalent.

1. $X$is coc-connected space.
2. The only coc-clopen set in $X$are $\emptyset\ and\ X$.
3. There exist no two disjoint coc-open sets $A\ and\ B$ such thatX= $A \cup B$.

Definition (3.4)
Let X be space.Two subsets U and V of space X called s-coc-separated if and only if $\overline{U}^{s-coc} \cap V = U \cap \overline{V}^{s-coc} = \emptyset$.

Definition (3.5)
Let X be a space and $\emptyset \neq A \subseteq X$.Then A is called s-coc-connected set if and only if is not union of any two s-coc-separated sets.
Not that a space (X,τ) is s-coc-connected if and only if (X,τ^{sk})connected.

Remark (3.2)
A set A is called s-coc-clopen if and only if it is s-coc-open and s-coc-closed.

Proposition (3.2)
Let X be space then the following statements are equivalent.
i. X is s-coc-connected space.
ii. The only s-coc-clopen set in X are $\emptyset\ and\ X$.
iii. There exist no two disjoint s-coc-open sets $A\ and\ B$ such that $= A \cup B$.

Proof
$(i) \longrightarrow (ii)$
Let X be s-coc-connected space.Suppose that D is s-coc-clopen set such that $D \neq \emptyset\ and\ D \neq X$, let $E = X - D$ since $D \neq X$ thenE$\neq \emptyset$. Since D is s-coc-open set. Then E is s-coc-closed But $\overline{D}^{s-coc} \cap E = D \cap E = \emptyset$ (since D is s-coc-clopen set and E is s-coc-closed).Hence $\overline{D}^{s-coc} \cap E = D \cap \overline{E}^{s-coc} = \emptyset$ then $D\ and\ E$ two s-coc-separated sets and $X = D \cup E$.Hence X is not s-coc-connected space, which is contradiction. Then the only s-coc-clopen sets in X are $\emptyset\ and\ X$.
$(ii) \longrightarrow (iii)$
Suppose the only s-coc-clopen set in the space are $\emptyset\ and\ X$, Let there exists two disjoint s-coc-open set $A\ and\ B$ such that $X = A \cup B$. Since $A = B^c$ then A is s-coc-clopen set.But $A \neq \emptyset$ and $A \neq X$, which is

contradiction. Hence there exist no two disjoint s-coc-open sets A and B such that $X = A \cup B$.

$(iii) \longrightarrow (i)$

Suppose that X is no s-coc-connected space.Then there exist two s-coc-separated sets A, B such that $X = A \cup B$ .Since $\overline{A}^{s-coc} \cap B = A \cap \overline{B}^{s-coc} = \emptyset$ and $A \cap B \subseteq \overline{A}^{s-coc} \cap B$.Thus $A \cap B = \emptyset$ since $\overline{A}^{s-coc} \subseteq B^c = A$.Then A is s-coc-closed set .

By the same way we can see that B is s-coc-closed set since $A^c = B$.Then A and B are two disjoint s-coc-open sets such that $X = A \cup B$.This is contradiction. Hence X is s-coc-connected space.

Proposition (3.3)

Every s-coc-connected space is connected space.

Proof clrar

But the convers is not true.

Example (3.1)

Let $X = \{1,2,3\}, \tau = \{\emptyset, X, \{1\}, \{2\}, \{1,2\}\}$ .It is clear that X is connected space.But X is not s-coc-connected space **(**Since $\{2\}, \{1,3\}$ are disjoint s-coc-open sets and $X = \{2\} \cup \{1,3\}$**)**

Proposition (3.4)

Let A be s-coc-connected set and D, E s-coc-separated sets.If $A \subseteq D \cup E$ then either $A \subseteq D$or $\subseteq E$.

Proof

Suppose A be s-coc-connected set and D, E are s-coc-separated sets and $A \subseteq D \cup E$, let $A \nsubseteq D$ and $A \nsubseteq E$. Suppose $A_1 = D \cap A \neq \emptyset$ and $A_2 = E \cap A \neq \emptyset$.Since $A \subseteq D \cup E$ then $(D \cup E) \cap A = A$.Thus $(D \cap A) \cup E \cap A = A$ then $A1 \cup A2 = A$.Since $A1 = D \cap A$ then $A1 \subseteq D$ then $\overline{A_1}^{s-coc} \subseteq \overline{D}^{s-coc}$.Since D, E are s-coc-separated sets then $\overline{D}^{s-coc} \cap E = \emptyset$.Then $\overline{A_1}^{s-coc} \cap A_2 = \emptyset$. Since $A_2 = E \cap A$ then $A_2 \subseteq E$ thus $\overline{A_2}^{s-coc} \subseteq \overline{E}^{s-coc}$.Then $A_1 \cap \overline{A_2}^{s-coc} = \emptyset$and $A = A_1 \cup A_2$.Then A is a union of two s-coc-separated sets A_1, A_2. Therefore A is not s-coc-connected sets.This contradiction then either $A \subseteq D$or $\subseteq E$.

Proposition (3.5)

Let X be a space such that any two elements x and y of X are contained in some s-coc-connected set of X. Then X is s-coc-connected.

Proof

Suppose X is not s-coc-connected.Then the union of two s-coc-separated sets A, B .Since A, B not empty sets then there exists a, b such that $a \in A, b \in B$, let F be s-coc-connected set of X which contains a, b . Therefore $F \subseteq A$ or $F \subseteq B$.Which is contradiction (Since $A \cap B = \emptyset$) .Therefore X is s-coc-connected space

Proposition (3.6)

If D is s-coc-connected set and $D \subseteq E \subseteq \overline{D}^{s-coc}$, then E is s-coc-connected.

Proof

Suppose E not s-coc-connected, then there exists two sets A, Bsuch that $\overline{A}^{s-coc} \cap B = A \cap \overline{B}^{s-coc} = \emptyset$ and $E = A \cup B$. Since $D \subseteq E = A \cup B$then either $D \subseteq A$ or $D \subseteq B$by proposition (3.2) .If $D \subseteq A$then $\overline{D}^{s-coc} \subseteq A$, Thus $\overline{D}^{s-coc} \cap B = \emptyset$, since $B \subseteq E = A \cup B \subseteq \overline{D}^{s-coc}$ then $\overline{D}^{s-coc} \cap B = B$. Therefore $B = \emptyset$ this contradiction hence E is s-coc-connected. By the same way we can get a contradiction if $D \subseteq B$ hence D is s-coc-connected

Proposition (3.7)

If space X contains a s-coc-connected set E such that $\overline{E}^{s-coc} = X$, then X is s-coc-connected.

Proof

Suppose E is s-coc-connected set in X such that $\overline{E}^{s-coc} = X$. Since $E \subseteq X = \overline{E}^{s-coc}$.Then byproposition (3. 5)**.**we get X is s-coc-connected.

Proposition (3.8)

If A is s-coc-connected set then $\overline{A}^{s-coc}$ is s-coc-connected.

Proof

Suppose A is s-coc-connected and $\overline{A}^{s-coc}$ is not s-coc-connected .Then there exists two sets D, E Such that $\overline{A}^{s-coc} = D \cup E$. Since $A \subseteq \overline{A}^{s-coc}$, then $A \subseteq D \cup E$, since A is s-coc-connected then by proposition (3.4) either $A \subseteq D$ or $A \subseteq E$.If $A \subseteq D$ then $\overline{A}^{s-coc} \subseteq \overline{D}^{s-coc}$ but $\overline{D}^{s-coc} \cap E = \emptyset$ hence $\overline{A}^{s-coc} \cap E = \Phi$, since $\overline{A}^{s-coc} = D \cup E$ then $E = \Phi$ this contradiction.If $A \subseteq E$ then $\overline{A}^{s-coc} \subseteq \overline{E}^{s-coc}$ but $D \cap \overline{E}^{s-coc} = \emptyset$hence $\overline{A}^{s-coc} \cap D = \emptyset$. Since $\overline{A}^{s-coc} = D \cup E$ then $D = \emptyset$ this contradiction.

Remark (3.3)

Let X be a topological space and $A \subseteq X$

1) If A is s-coc-connected set in X then $\overline{A}$need not to be s-coc-connected.
2) If A is connected set then $\overline{A}^{s-coc}$ need not to be connected set.
3) If $\overline{A}^{s-coc}$ connected set then A need not to be connected.

Examples (3.2)

1) Let $X = \{1,2,3\}, \tau = \{\emptyset, X\}$. Then $\tau^{sk} = \{\emptyset, X, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}\}$.If $A = \{1\}$ then A is s-coc-connected. But $\overline{A} = X = \{1\} \cup \{2,3\}$, since $\{1\}, \{2,3\}$ are two s-coc-separated sets then $\overline{A}$ is not s-coc-connected**.**

2)Let$X = R$ and $\tau = \{\emptyset, R, Z, Z_e, Z_\circ\}$, $if A = \{1,2\}$is not union of two separated open sets.Then A is connected . But $\overline{A}^{s-coc} = Z = Z_e \cup Z_\circ$and $Z_e, Z_\circ$ are separated open sets.

3) Let $X = Z$, $\tau = \{\emptyset, R, Z, Z_e, \{1\}, \{3\}, \{1,3\}, Z_e \cup \{1\}, Z_e \cup \{3\}, Z_e \cup \{1,3\}\}$.

If $A = \{1,3\} = \{1\} \cup \{3\}$ then A is union of two disjoint open sets.Thus Adisconnected. But $\overline{A}^{s-coc} = Z_\circ$ is not union of two disjoint s-coc-open sets.Then $\overline{A}^{s-coc}$ connected set.

Proposition (3.9)
Every s-coc-connected space is coc-connected.

Proof
Let X is s-coc-connected set and not coc-connected then there exists two sets U, V such that $X = U \cup V$. Since U, V coc-open sets then U, V are s-coc-open sets.Then X is union of two s-coc-separated sets.Thus X is s-coc-disconnected.This conduction. Therefore X is coc-connected.But the converse is not true for the following example.

Example (3.3)
Let $X = \{1, 2, 3, ...\}$, $\tau = \{\emptyset, X, \{1, 2\}\}$ **.** It is clear that X is coc –connected but not s-coc-connected

Proposition (3.10)
The s-coc-continuous, onto image of s-coc-connected space is connected .

Proof
Let $f: (X, \tau) \longrightarrow (Y, \acute{\tau})$ be s-coc-continuous, onto function and X is s-coc-connected.To prove Y is connected. Let Y not connected then $Y = A \cup B$ suchthat $A \neq \emptyset$ $B \neq \emptyset$ and $A \cap B = \emptyset$ and A, B are open sets, hence $f^{-1}(Y) = f^{-1}(A \cup B)$ since f onto.Then $X = f^{-1}(A) \cup f^{-1}(B)$.Since A, B are open sets in X and f s-coc-continuous.Then $f^{-1}(A), f^{-1}(B)$ are s-coc-open sets in Y.Since $A \cap B = \emptyset$ thus $f^{-1}(A) \cap f^{-1}(B) = \emptyset$ and $A \neq \emptyset, B \neq \emptyset$.Hence X s-coc-disconnected space this is contradiction. Hence Y is connected space.

Remark (3.4)
The s-coc-continuous image of s-coc-connected space need not to be s-coc-connected

Example (3.4)
Let $X = \{1, 2, 3, ...\}, \tau = \{\emptyset, X\}$ and $f: (X, \tau) \longrightarrow (Y, \acute{\tau})$ s-coc-continuous function and $Y = \{a, b\}$, $\acute{\tau} = \{\emptyset, Y\}$ $f(x) = \begin{cases} a & if\ x = 1 \\ b & if\ x \neq 1 \end{cases}$. Since $\{a, b\} = \{a\} \cup \{b\}$ and $\{a\} \cap \{b\} = \emptyset$.Since $\{a\}, \{b\}$ are s-coc-open sets.Then Y is not s-coc-connected , Y open set in Y and $f^{-1}(Y) = X$ not union of two disjoint s-coc-open sets. Thus X s-coc-connected.

Proposition (3.11)
The s-coć-continuous image, onto of s-coc-connected space is s-coc-connected.

Proof
Let $f: (X, \tau) \longrightarrow (Y, \acute{\tau})$ s-coć-continuous, onto function and X s-coc-connected. To prove Y is s-coc-connected. Suppose Y is not s-coc-connected.Then $Y = A \cup B$ such that A, B s-coc-open sets and, $B \neq \emptyset, A \cap B = \emptyset$. Hence$f^{-1}(Y) = f^{-1}(A \cup B)$. Since f onto then $X = f^{-1}(A) \cup f^{-1}(B)$.Since A, B s-coc-open sets and f s-coć-continuous thus $f^{-1}(A), f^{-1}(B)$ s-coc-open sets in X and $f^{-1}(A) \cap f^{-1}(B) = \emptyset$.Then X is not s-coc-connected. This contradiction.then Y is s-coc-connected.

Proposition (3.12)
If $f: X \longrightarrow Y$ s-coc´-open and Y s-coc-connected, then X s-coc-connected.

Proof
Let Y s-coc-connected and X is not s-coc-connected. To get contradiction, since is not s-coc-connected then $X = W_1 \cup W_2$ such that W_1, W_2 disjoint s-coc-open sets in X. Since f is s-coc´-open then $f(W_1), f(W_2) disjoints$-coc-open sets in Y and $f(X) = f(W_1 \cup W_2) = f(W_1) \cup f(W_2)$ then $Y = f(W_1) \cup f(W_2)$.Then Y is not s-coc-connected.This contradiction. Then X is s-coc-connected.

Remark (3.5)
If $f: X \longrightarrow Y$ s-coc´-homeomorphism then X is s-coc-connected if and only if Y is s-coc-connected

Proposition (3.13)
Let X be space and $Y = \{0, 1\}$ have the discrete topology X is s-coc-connected if and only if there is no s-coc-continuous function from X to Y.

Proof
Let $f: (X, \tau) \longrightarrow (Y, \acute{\tau})$ s-coc-continuous onto function.Then there is exists $x, y \in X$ such that $x \neq y$ and $f(x) = 0 and$ $f(y) = 1$ then $f^{-1}(0) = x = A$ and $f^{-1}(1) = y = B$.Then A, B s-coc-open sets and $X = A \cup B$ such that $A \cap B = \emptyset, A \neq \emptyset$, $B \neq \emptyset$.Then X is s-coc-disconnected
Conversely:
Let there is no s-coc-continuous onto function, let X is s-coc-disconnected.Then $X = A \cup B$ such that $A \neq \emptyset$, $B \neq \emptyset$ and A, B are $disjoint$ s-coc-open sets. Define $g: (X, \tau) \longrightarrow (Y, \acute{\tau})$ such that $g(x) = \begin{cases} 0 & \forall\, x \in A \\ 1 & \forall\, x \in B \end{cases}$, then$f^{-1}(0) = A$, $f^{-1}(1) = B$.Then f s-coc-continuous, this contradiction. Thus X is s-coc-connected.

Proposition (3.14)
If f is s-coc-continuous onto and g is continuous function and onto then $g \circ f$ image of s-coc-connected set is connected
Proof

Let $f: (X, \tau) \longrightarrow (Y, \acute{\tau})$ and $g: (Y, \acute{\tau}) \longrightarrow (W, \tau^*)$, let A s-coc-connected set in$X$, $(g \circ f)(A) = g\big(f(A)\big)$.Since f is s-coc-continuous then $f(A)$connected in Y by proposition (3.9). Since g is continuous then $g\big(f(A)\big)$ connected set in W.Then $(g \circ f)(A)$ connected set.

Proposition (3.15)
If f is s-coć-continuous onto and g is s-coc-continuous onto function then $g \circ f$ image of s-coc-connected set is connected

Proof
Let A s-coc-connected set in$X$, $(g \circ f)(A) = g\big(f(A)\big)$.Since f is s-coć-continuous then $f(A)$ is s-coc-

connected.Then $(g \circ f)(A)$ connected set by proposition (3.10) and proposition (3.2).

Proposition (3.16)
If $f: X \longrightarrow Y$ super s-coc-open function and Y connected. Then X s-coc-connected.

Proof
Let X is not s-coc-connected. To get contradiction.Since X is not s-coc-connected then there exists A *and* B disjoint s-coc-open sets such that X= $A \cup B$. Since f super s-coc-open then $f(A), f(B)$ are open sets in Y. Since $A \cap B = \emptyset$then $f(A) \cap f(B) = f(A \cap B) = \emptyset$. Then $f(A), f(B)$ disjoint $f(A) \cup f(B) = f(A \cup B) = f(X) = Y$ Then Y is not connected space.This contradiction then X is s-coc-connected space.

Definition (3.8) [5]
A space (X, τ) is said to be locally connected if for each point $x \in X$ and each open set U such that $x \in U$ there is a connected open set V such that $x \in V \subseteq U$.

Definition (3.9)
A space (X, τ) is said to be s-coc-locally connected if for each point $x \in X$ and each s-coc-open set U such that $x \in U$ there is s-coc-connected open set V such that $x \in V \subseteq U$.

Proposition (3.17)
Every s-coc-locally connected space is locally connected space.

Proof
Let X is s-coc-locally connected, let $x \in X$ and U open set in X such that $x \in U$.Then there is s-coc-connected set V such that $x \in V \subseteq U$. Since every s-coc-connected is connected by proposition (3.2). Then V is connected open set in X such that $\in V \subseteq U$, hence X is locally connected space. But the converse is not true for the following example.

Example (3.5)
Let $X = \{1,2,3,\dots\}$, $\tau = \{\emptyset, X, \{2,3\}\}$Then τ^{sk} *is* discrete Topology in X**.** Since $1 \in X$ and $\{1,2\}$ s-coc-open set $\{1\} \in \{1,2\}$, since there is no s-coc-connected open set V such that $1 \in V \subseteq \{1,2\}$ **.**Thus X is not s-coc-locally connected.

Not that:
if$f: (X, \tau) \longrightarrow (Y, \tau^*)$ super s-coc-open function and Y s-coc-locally connected but X is not s-coc-locally connected for example.

Example (3.6)
Let $X = \{a,b,c,d\}, \tau = \{\emptyset, X\}$, $Y = \{0,1\}, \tau^*$ *discrete topology* . Define$f(x) = \begin{cases} 0 \ \ if \ \ x \in \{a,b\} \\ 1 \ \ if \ \ x \in \{c,d\} \end{cases}$,
Sinceτ^{sk} in X is discrete and $f(A)$ open in Y forall A s-coc-open in$X$. Then f super s-coc-open and Y s-coc-locally connected but $a \in X$ and $\{a,b\}$ s-coc-open set such that $a \in \{a,b\}$ and there exists no s-coc-connected open set V in X such that$a \in V \subseteq \{a,b\}$. Then X is not s-coc-locally connected.

Definition (3.10) [7]
A space (X, τ) is said to be coc-locally connected if for each point $x \in X$ and each coc-open set U such that $x \in U$ there is coc-connected open set V such that $x \in V \subseteq U$.

Proposition (3.18)
Every s-coc-locally connected space is coc-locally connected space.

Proof
Let X be s-coc-locally connected space and$x \in X, U$ open set in X such that $x \in U$.Since every coc-open set is s-coc-open set. Then U s-coc-open set. Since X s-coc-connected then there is V s-coc-connected open set such that $x \in V \subseteq U$ Thus V is coc-connectedopen by proposition (3.8).Then there is V coc-connected open set such that x$\in V \subseteq U$. Then X is coc-locally connected space.

But the converse is not true for the following example

Example (3.7)
Let $X = \{1,2,3,4,\dots\}, \tau = \{\emptyset, X, \{1\}\}$. Since$1 \in X, \{1\}$ coc-open set *and* $1 \in U = \{1\}$Since$V = \{1\}$ coc-connected open set and $1 \in V \subseteq U$. Since$x = 2,3,4,\dots \in X, X$ coc-open set *suchthat* $x \in X = U$, since $V = X$ coc-connected open set and $x \in X \subseteq X$Thus$X$ coc-locally connected.But X is not s-coc-locally connected. Since s-coc-open sets are discrete Topology. Since $5 \in X$ and U={4, 5} s-coc-open set, $5 \in U$ and there is no s-coc-connected open set V such that $5 \in V \subseteq \{4,5\} = U$.

Remark (3.6)

1) If (X, τ) is s-coc-locally connected space, then it need not to be s-coc-connected.
2) If (X, τ) is s-coc-connected space, then it need not to be s-coc-locally connected.

Examples (3.8)
1) Let $X = \{1,2,3,4,\dots\}$ *and* τ *discete Topology*. Then s-coc-connected discrete topology Thus *for all* $x \in X$ *and for all* U s-coc-open set, $x \in U$ *there is* V s-coc-connected open set such that $x \in V \subseteq U$.Then X s-coc-locally connected. But $X = \{1\} \cup \{1\}^c$ and $\{1\} \cap \{1\}^c = \emptyset$ such that $\{1\}, \{1\}^c \neq \emptyset$ *and* $\{1\}, \{1\}^c$ are s-coc-open sets. Thus X not s-coc-connected.

2) Let $X = \{1,2,3,4,\dots\}, \tau = \{\Phi, X, \{1,2\}^c\}$ *then* $\tau^{sk} = \{A \subseteq X \ni A \text{infinite}\}$.Then X is not union of two disjoints-coc-open sets. Thus X is s-coc-connected.But$5 \in X, U = \{1,2,5,6,7,\dots\}$ s-coc-open set, $5 \in U$ and there is no s-coc-connected open set V such that $5 \in X \subseteq U$.Thus X is not s-coc-locally connected.

Proposition (3.19)
The s-coc-continuous and open onto image of s-coc-locally connected space is locally connected.

Proof

Let $f:(X,\tau) \longrightarrow (Y,\acute{\tau})$ be s-coc-continuous, open onto function and (X,τ) is s-coc-locally connected. To prove $(Y,\acute{\tau})$ is locally connected, let $y \in Y$ and U open set in Y such that $\in U$. Since f s-coc-continuous then $f^{-1}(U)$ is s-coc-open sets in Y. Since X is s-coc-locally connected then there is V s-coc-connected open set such that $x \in V \subseteq f^{-1}(U)$. Since f open then $f\ (V)$ open in Y, since V is s-coc-connected then $f\ (V)\ connected$ by proposition (3.9). Thus $(x) = y \in f\ (V)\ \subseteq U$. Then Y is locally connected.

Remark (3.7)

The s-coc-continuous image of s-coc-locally connected need not to be s-coc-locally connected.

Example (3.9)

Let $X = \{1,2,3,\dots\}, \tau\ discrete\ topology, Y = \{a,b\}, \tau^{'} = \{\emptyset, Y, \{a\}\}$. Define $f:(X,\tau) \longrightarrow (Y,\tau^{'})\, f(x) = \begin{cases} a & if\ x = 1 \\ b & if\ x \neq 1 \end{cases}$, since open sets in Y are Y, $\{a\}$ and $f^{-1}(Y) = X, f^{-1}(\{a\}) = \{1\}$ s-coc-open in X.Thus f s-coc-continuous and X s-coc-locallyconnected. But $b \in Y\ and\ \{b\}$ s-coc-open set in Y such that $b \in \{b\}$ and there exists no s-coc-connected open set V such that b$\in V \subseteq \{b\}$. Thus Y is not s-coc-locally connected.

Proposition (3.20)

The s-coć-continuous, open onto image of s-coc-locally connected space is s-coc-locally connected.

Proof

Let $f:(X,\tau) \longrightarrow (Y,\acute{\tau})$ be s-coć-continuous, open onto function and (X,τ) is s-coc-locally connected. To prove $(Y,\acute{\tau})$ is s-coc-locally connected. Let $y \in Y$ and U s-coc-open set in Y such that y $\in U$. Since f onto and $y \in Y$ there is $x \in X$ such that $f(x) = y$. Since f s-coć-continuous then $f^{-1}(U)$ s-coc-open sets in $\boldsymbol{X}$. Since X is s-coc-locally connected then there is V s-coc-connected open set in X such that $\in V \subseteq f^{-1}(U)$. Since f open then $f\ (V)$ open in Y and connected by proposition (3.10). Then y= $f(x) \in f\ (V)\ \subseteq U$. Thus Y is s-coc-locally connected.

Definition (3.11) [12]

A space (X,τ) is said to be extremely disconnected if the closure of every open subset of the X is open in X.

Definition (3.12)

A space (X,τ) is said to be s-coc-extremely disconnected if the closure of every open subset of the X is s-coc-open in X .

Remark (3.8)

Every extremely disconnected space is s-coc-extremely. But the convers is not true.

Example (3.10)

Let $X = \{1,2,3,4,\dots\}, \tau = \{\emptyset, X, \{1\}, \{2\}, \{1,2\}\}$.Then τ^{sk} is discrete topologyon X. Then every subset in X is s-coc-open and s-coc-closed set.Thus if A open in X, then $\overline{A}$ is s-coc-open set. Thus X is s-coc-extremely disconnected. But $A = \{1\}$ open set and $\overline{A} = \{1,3,4,5,\dots\}$ not open. Then X is not extremely disconnected.

Proposition (3.21)

For a topological space (X,τ) if the closure of every s-coc-open set is s-coc-open, then X is s-coc-extremely disconnected.

Proof

Let U is open set of X. Then U is s-coc-open set. Since the closure of s-coc-open set is s-coc-open set. Thus X is s-coc-extremely disconnected.

Proposition (3.22)

If $X\ and\ Y$ s-coc-extremely disconnected then $X \times Y$ s-coc-extremely disconnected.

Proof

Let $W = A \times B$ open in $X \times Y$. Then A, B open in X, Y.Since X, Y s-coc-extremely then $\overline{A}, \overline{B}$ s-coc-open in $X, Y\ \ then\ \overline{W} = \overline{A \times B} = \overline{A} \times\ \overline{B}$ s-coc-open set by proposition (1.3).Then $\overline{A} \times\ \overline{B}$ s-coc-open setin $X \times Y$. Then $X \times Y$ s-coc-extremely disconnected.

Remark (3.9)

If X s-coc-extremely disconnected then X need not to be s-coc-connected for example .

Example (3.11)

Let $X = R$ and U usual Topology on R.Since $R = (-\infty,0)\cup[0,\infty)$ and$(-\infty,0)\cup[0,\infty) = \emptyset$ and $(-\infty,0), [0,\infty)$ s-coc-open sets .Thus (R,U) s-coc-disconnected.But for every (a,b) open set in R.Thus $\overline{(a,b)} = [a,b]$ is s-coc-open set in R. Therefore X is s-coc-extremely.

Proposition (3.23)

If X is s-coc-connected then X is not s-coc-extremely disconnected.

Proof

Let X s-coc-connected and X is s-coc-extremely disconnected. To get contradiction.Then for all A open set we get $\overline{A}$ s-coc-open. Since $\overline{A}$ closed set then $\overline{A}$ s-coc-closed.Then X is not s-coc-connected by proposition (3.2) (if X is s-coc-connected then the only s-coc-clopen sets are$\emptyset, X$). Therefore X is not s-coc-extremely disconnected

Not that

If X is s-coc-extremely disconnected then X need not to be s-coc-locally connected for the following example.

Example (3.12)

Let $X = \{1,2,3,\dots\}, \tau = \{\Phi, X, \{1,2\}\},\ since\ \{1,2\}$ open set in X and $\overline{\{1,2\}} = X$ s-coc-open set. Then X is s-coc-extremely disconnected.But $1 \in \{1,2\}$ open and there is no V s-coc-connected open set such that $1 \in V \subseteq \{1,2\}$. Then X is not s-coc-locally connected.

Definition (3.13)
Let X be a space, $A \subseteq X$ is called s-coc-dense set in X if and only if $\overline{A}^{s-coc} = X$

Definition (3.14) [8]
Aspace (X, τ) is said to be hyper connected if every non-empty open subset of X is dense.

Definition (3.15)
Aspace (X, τ) is said to be s-coc-hyper connected if every non-empty s-coc-open subset of X is s-coc-dense.

Proposition (3.24)
Every s-coc-hyper connected space is hyper connected.

Proof
Let X s-coc-hyper connected space. Then for all s-coc-open set of X is s-coc-dense in$X$.Then $\overline{A}^{s-coc} = X$. To prove $\overline{A} = X$ since $\overline{A} \subseteq X$ … (1), let $x \in X$ then $x \in \overline{A}^{s-coc}$ .Since $\overline{A}^{s-coc} \subseteq \overline{A}$ then $x \in \overline{A}$ then $X \subseteq \overline{A}$ … (2) . Therefore $\overline{A} = X$ then X is hyper connected space.
But the convers is not true.

Example (3.13)
Let$X = \{a, b, c\}$ *and* $\tau = \{\emptyset, X\}$ then $\tau^{sk} = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$.Since$\overline{A} = X$ for all $A \subseteq X$ .But $\overline{A}^{s-coc} = A \neq X$ .Then X is not s-coc-hyper connected but hyper connected

Proposition (3.25)
Every s-coc-hyper connected space is s-coc-connected.

Proof
Let X s-coc-hyper connected space and X is not s-coc-connected. Then there exists A s-coc-clopen set such that $A \neq \emptyset$ and $A \neq X$ hence $A = \overline{A}^{s-coc}$ .This contradiction (since$\overline{A}^{s-coc} = X$).Then X is s-coc-connected space.

Proposition (3.26)
Every s-coc-hyper connected is s-coc-extremely disconnected.

Proof
Let X s-coc-hyper connected space. Then for all A s-coc-open is s-coc-dense then$\overline{A}^{s-coc} = X$. Then $X \subseteq \overline{A}$. Since$\overline{A} \subseteq X$ then$\overline{A} = X$. Since X s-coc-open set. Then for all A open set we get the closure is s-coc-open set. Then X s-coc- extremely disconnected.
But the converse is not true for example

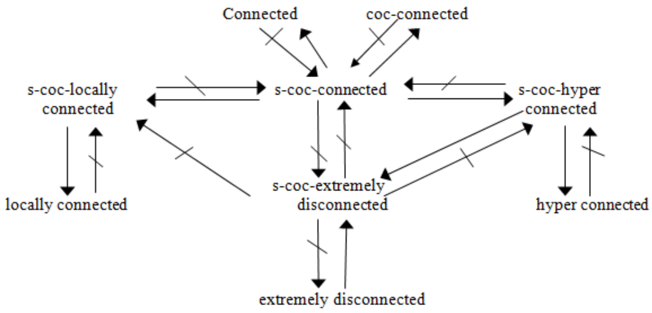
Example(3.14)
Let $X = \{1, 2, 3\}$ *and* $\tau = \{\emptyset, X, \{1\}\}$ *then* $\tau^{sk} = \{\emptyset, X, \{1\}, \{2\}\{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}\}$. If $A = \{2\}$ then $\overline{A}^{s-coc} = \{2, 3\} \neq X$. Then A not s-coc-dense we get X is not s-coc-hyper connected.But $\overline{B}$ is s-coc-open for all B open set in X.Then X s-coc- extremely disconnected

Not that
The s-coc-continuous image of the s-coc-hyper connected need not to be s-coc-hyper connected.

Example (3.15)
Let$X = Z, \tau = \{\emptyset, Z, Z_{\circ}\}$ topology on Z and $Y = \{a, b\}$, $\tau^* = \{\emptyset, Y, \{a\}\}$ topology on $Y, f: (X, \tau) \longrightarrow (Y, \tau^*)$ function defined by $f(x) = \begin{cases} a & \text{if } x \in Z_{\circ} \\ b & \text{if } x \in Z_e \end{cases}$.Then f s-coc-continuous and

$$\tau^{sk} = \{\emptyset, Z, Z_{\circ}, Z_{\circ} \cup \text{any set}, Z_{\circ} - \text{finite set}\}$$

$\overline{Z_{\circ}}^{s-coc} = Z$, $\overline{Z_{\circ} \cup \text{any set}}^{s-coc} = Z, \overline{Z_{\circ} - \text{finite set}}^{s-coc} = Z$.
Then Z s-coc-hyper connected space.But $\overline{\{a\}}^{s-coc} = \{a\} \neq Y$. Then Y is not s-coc-hyper connected space.



Reference

[1] N.K. Ahmed "On Some Application of special subset of Topological spaces" M.S.C. Thesis, University of salahaddin, College of science, 1990.
[2] Bourbaki N., Elements of Mathematics "General topology " Chapter 1-4, Spring Vorlog, Belin, Heidelberg, New- York, London, Paris, Tokyo 2 nd Edition (1989).
[3] S.G. Crossley and S.K.Hildebr and "On Somi Closure" Taxas J. Sci., 22, (1971).
[4] J. Dugundji, Topology, Allyn and Bacon Baston, 1966.
[5] M. C. Gemignani, "Elementary Topology" Reading Mass. Addison-wesley Publishing Co.Inc., 1967.
[6] H. Z. Hdieb "ω-closed mapping", Rev.Colombo Math.16 (1-2) (1982), 65-78.
[7] F. H. Jasim "On Compactness via Cocompact open sets" M.SC. Thesis University of Al-Qadissiya, college of Mathematics and computer science, 2014.
[8] N.J. Al-Malaki"Some kinds of Weakly Connected and pair Wise Connected space" M.SC. University of Baghdad College of Education / Abn Al-Latham, 2005.
[9] N. Levine "Semi-open sets and Semi Continuity in Topological space "Amer Math. Monthy , 70, (1963).
[10] A. G. Naoum and R.A.H. Al-Abdulla " On parcompact space using semi-open sets ''Al-Qadisiya journal for computer science and Mathematics, vol 2, No.1 (2010).
[11] S.Alghour and S.Samarah"Cocompact open sets and Continuity" Abstract and applied analysis, Volume 2012, Article ID 548612, 9 Pages, 2012.
[12] M.H.Tone, Applications of the theory of Boolean Rings To general topology, Trans. AmerM.OC.41(3) (1937) 375-481