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Approximation of Systems of Volterra Integro-Differential Equations Using the New Iterative Method

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Abstract: ***In this paper, the new iterative method with a reliable algorithm is applied to the systems of Volterra integro-differential equations. The method is useful for both linear and nonlinear equations. By using this method, the solutions are obtained in series form. Two linear and one nonlinear system of the equations are given to verify the reliability and efficiency of the method. Beside this, the comparison of the exact solution with the approximated solution by the method is illustrated by the graphs.***

Keywords: New Iterative Method, System of Volterra Integro-Differential Equations, Exact solutions.

1. Introduction

Integro-differential equations have found applications in engineering, physics, chemistry, biology and insurance mathematics [1-3]. Several techniques including Chebyshev polynomial method [4], hybrid Legendre functions [5], the Taylor collocation method [6], the differential transformation method [7] , the variational iteration method [8] , the Beseel collocation method [9], [10] the homotopy perturbation method [11], [12] and Taylor series method [13] have been used to investigate integro-differential equations.

In recent review article, Khan [14] have obtained the solutions of the systems of third order nonlinear integro-differential equations by Variational iteration algorithm-II method. Yuzbasi and Isnailov [15] applied the differential transformation method to system of Volterra integro-differential equations with propositional delay. Recently, Daftardar-Gejji and Jafari [16] proposed a reliable method for solution of functional equations called the new iterative method. Hemeda [17] implements the new iterative method to nth order linear and nonlinear integro-differential equations. The elegance of the new iterative method can be attributed to its simplistic approach in finding the semi-analytical solution of the system of nonlinear integro-differential equation of the form:

$$y_1^{(m)}(x) = f_1\left(\begin{matrix} x, y_2(t), \dots, y_2^{(m)}(t), y_3(t), \dots, y_3^{(m)}(t), \dots, \\ y_n(t), \dots, y_n^{(m)}(t) \end{matrix}\right)$$
$$+ \int_0^x K_1\left(x, t, y_1(t), \dots, y_1^{(m)}(t), \dots, y_n(t), \dots, y_n^{(m)}(t)\right) dt$$
$$y_2^{(m)}(x) = f_2\left(\begin{matrix} x, y_1(x), \dots, y_1^{(m)}(x), y_3(x), \dots, y_3^{(m)}(x), \dots, \\ y_n(x), \dots, y_n^{(m)}(x) \end{matrix}\right)$$
$$+ \int_0^x K_2\left(x, t, y_1(t), \dots, y_1^{(m)}(t), \dots, y_n(t), \dots, y_n^{(m)}(t)\right) dt, \qquad (1)$$
$$\dots$$
$$y_n^{(m)}(x) = f_n\left(\begin{matrix} x, y_1(x), \dots, y_1^{(m)}(x), y_2(x), \dots, y_2^{(m)}(x), \dots, \\ y_{n-1}(x), \dots, y_{n-1}^{(m)}(x) \end{matrix}\right)$$
$$+ \int_0^x K_n\left(x, t, y_1(t), \dots, y_1^{(m)}(t), \dots, y_n(t), \dots, y_n^{(m)}(t)\right) dt,$$

In systems (1), m is order of derivatives and the continuous several variables functions and f_i and K_i , $i = 1,2,3, \dots, n$ are given, the solutions to be determined are $y_i(x)$, $i = 1,2,3, \dots, n$

2. Basic Idea of New Iterative Method

To describe the idea of the new iterative method (NIM), we consider the following general formulation by Dafatardar-Gejji and Jafari (2006). Consider the nonlinear functional equation:

$$N(y(x)) \qquad (2)$$

where N is a nonlinear operator from a Banach space $B \to B$ and is a known function. We are looking for a solution $y(x)$ of (2) of having the series form:

$$y(x) = \sum_{i=0}^{\infty} y_i \qquad (3)$$

The nonlinear operator can be decomposed as follows:

$$N\left(\sum_{i=0}^{\infty} y_i\right) = N(y_0) + \sum_{i=1}^{\infty}\left\{N\left(\sum_{j=0}^{i} y_j\right) - N\left(\sum_{j=0}^{i-1} y_j\right)\right\} \qquad (4)$$

From Eq. (3) and (4), Eq. (2) is equivalent to

$$N\left(\sum_{i=0}^{\infty} y_i\right) = f(x) + N(y_0) + \sum_{i=1}^{\infty}\left\{N\left(\sum_{j=0}^{i} y_j\right) - N\left(\sum_{j=0}^{i-1} y_j\right)\right\} \qquad (5)$$

we define the recurrence relation:

$$\left.\begin{aligned} y_0 &= f(x) \\ y_1 &= N(y_0) \\ y_2 &= N(y_0 + y_1) - N(y_0) \\ &\dots \\ y_{m+1} &= N(y_0 + y_1 + \dots + y_m) \\ &- N(y_0 + y_1 + \dots + y_{m-1}), m = 1,2, \dots \end{aligned}\right\} \qquad (6)$$

$$y_0 + y_1 + \dots + y_{m+1} = N(y_0 + y_1 + \dots + y_m), m = 1,2, \dots, \qquad (7)$$

$$y(x) = f(x) + \sum_{i=0}^{\infty} y_i \qquad (8)$$

If N is a contraction, i.e.

$$\|N(x) - N(y)\| \le k\|x - y\|, 0 < k < 1,$$

Then,

$$\|y_{m+1}\| = \|N(y_0 + y_1 + \dots + y_m) - N(y_0 + y_1 + \dots + y_{m-1})\|$$

$$\le k\|y_m\| \le \dots \le k^n\|y_0\|, \ m = 0,1,2,\dots,$$

and the series $\sum_{i=0}^{\infty} y_i$ absolutely and uniformly converges to solution of Eq. (1) [18], which is unique, in view of Banach fixed point theorem[19]. The k-term approximate solution of Eq. (2) and (3) is given by $\sum_{i=0}^{k-1} y_i$.

3. New Iterative Method for the System

For simplicity, let us rewrite the system of nonlinear Volterra integral equations in Eq. (1) above in vector form as:

$$\boldsymbol{y}^{(m)}(x) = \boldsymbol{f}(x) + \int_0^x (\boldsymbol{K}(x,t,y_1^{(m)}, y_2^{(m)}, \dots, y_n^{(m)})\, dt \tag{9}$$

where

$$\boldsymbol{f}(x) = [f_1(x),\ f_2(x), \dots, f_n(x)]^T,$$

$$\boldsymbol{y}^{(m)}(x) = [y_1^{(m)}, y_2^{(m)}, \dots, y_n^{(m)}]^T,$$

$$\mathbf{K} = [K_1, K_2, \dots, K_n]$$

In view of the new iterative method, the system of Volterra integro-differential equation in Eq. (9) is equivalent to the system of integral equation:

$$\boldsymbol{y}(x) = (x) + I_x^m\left(\int_0^x (\boldsymbol{K}(x,t,y_1^{(m)}, y_2^{(m)}, \dots, y_n^{(m)})\, dt\right) \tag{10}$$

$$= \boldsymbol{y}_0(x) - N(\boldsymbol{y}) \tag{11}$$

where the zero solution $\boldsymbol{y}_0(x)$ is the solution of the system of nth-order integro-differential equation and

$$\frac{d^n y_0}{dx^n} = g(x) - \boldsymbol{f}(x)\boldsymbol{y}(x), \frac{d^n y_0}{dx^n} = \alpha_r, \ \ r = 0,1,2,\dots,n-1. \tag{12}$$

$$\boldsymbol{y}_0(x) = [y_{10}(x),\ y_{20}(x), \dots, y_{n0}(x)]^T,$$

$$\boldsymbol{y}(x) = [y_1(x), y_2(x), \dots, y_n(x)]^T$$

$$\boldsymbol{f}(x) = [f_1(x),\ f_2(x), \dots, f_n(x)]^T,$$

$$\mathbf{K} = [K_1, K_2, \dots, K_n]$$

Therefore,

$$N(\boldsymbol{y}) = I_x^n\left(\int_0^x \left(\boldsymbol{K}(x,t)\boldsymbol{y}^{(q)}(t)\boldsymbol{y}^{(m)}(t)\right) dt\right) \tag{13}$$

where I_x^n is an nth-order integral operator with respect to x.

In vector notation, the new iterative algorithm for (13) is:

$$\boldsymbol{y}_1(x) = I_x^n\left(\int_0^x \left(\boldsymbol{K}(x,t) y_{10}{}^{(q)}(t) y_{20}{}^{(m)}(t)\right) dt\right)$$

$$\boldsymbol{y}_2(x) = I_x^n\Big(\int_0^x \big(\boldsymbol{K}(x,t)(y_{10}{}^{(q)}(t) + y_{11}{}^{(q)}(t)(y_{20}{}^{(m)}(t) + y_{21}{}^{(q)}(t))\big)dt\Big) - I_x^n\left(\int_0^x \left(\boldsymbol{K}(x,t) y_{10}{}^{(q)}(t) y_{20}{}^{(m)}(t)\right) dt\right)$$

$$\dots$$

$$\boldsymbol{y}_r(x) = I_x^n\Big(\int_0^x \big(\boldsymbol{K}(x,t)(y_{10}{}^{(q)}(t) + \dots + y_{1,r}{}^{(q)}(t))(y_{20}{}^{(m)}(t) + \dots + y_{2,r}{}^{(q)}(t)\big)dt\Big) - I_x^n\Big(\int_0^x \big(\boldsymbol{K}(x,t)(y_{10}{}^{(q)}(t) + \dots + y_{1,r-1}{}^{(q)}(t))(y_{20}{}^{(m)}(t) + \dots + y_{2,r-1}{}^{(q)}(t)\big)dt\Big)$$

where:

$$\boldsymbol{y}_1 = [y_{11}, y_{21}, \dots, y_{n1}]^T,$$

$$\dots$$

$\boldsymbol{y}_r = [y_{1r}, y_{2r}, \dots, y_{nr}]^T$, and

$$\boldsymbol{K} = [K_1, K_2, \dots, K_n]^T,$$

4. Illustrative Examples

Example 1.

Consider the system of first-order linear Volterra integro-differential equation:

$$y_1'(x) = 1 + x - \frac{1}{3}x^3 + \int_0^x \big((x-t)y_1(t) + (x-t)y_2(t)\big)\, dt \tag{14}$$

$$y_2'(x) = 1 - x - \frac{1}{12}x^4 + \int_0^x \big((x-t)y_1(t) - (x-t)y_2(t)\big)\, dt$$

with the initial conditions:

$y_1(0) = 0,\ y_1'(0) = 0$ and

$$y_2(0) = 0,\ y_2'(0) = 0$$

The system of the integro-differential equation (14) is equivalent to the system of integral equation:

$$y_{10}(x) = x + \frac{1}{2}x^2 - \frac{1}{12}x^4 + I_x'\left[\int_0^x \big((x-t)y_1(t) + (x-t)y_2(t)\big)dt\right]$$

$$y_{20}(x) = x - \frac{1}{2}x^2 - \frac{1}{60}x^5 + I_x'\left[\int_0^x \big((x-t)y_1(t) - (x-t)y_2(t)\big)dt\right]$$

Let

$$N_1(y) = I_x'\left[\int_0^x \big((x-t)y_1(t) + (x-t)y_2(t)\big)dt\right]$$

$$N_2(y) = I_x'\left[\int_0^x \big((x-t)y_1(t) - (x-t)y_2(t)\big)dt\right]$$

we obtain easily the following first few components of the new iterative method solution.

The first five terms are:

$$y_{10}(x) = x + \frac{1}{2}x^2 - \frac{1}{12}x^4$$

$$y_{20}(x) = x - \frac{1}{2}x^2 - \frac{1}{60}x^5$$

$$y_{11}(x) = I_x'\left[\int_0^x \big((x-t)y_{10}(t) + (x-t)y_{20}(t)\big)dt\right]$$

$$= -\frac{1}{20160}x^8 - \frac{1}{2520}x^7 + \frac{1}{12}x^4$$

$$y_{21}(x) = I_x'\left[\int_0^x \big((x-t)y_{10}(t) - (x-t)y_{20}(t)\big)dt\right]$$

$$= \frac{1}{20160}x^8 - \frac{1}{2520}x^7 + \frac{1}{60}x^5$$

$$y_{12}(x) = I_x'\left[\int_0^x \big((x-t)(y_{10}(t) + y_{11}(t)) + (x-t)(y_{20}(t) + y_{21}(t))\big)dt\right] - I_x'\left[\int_0^x \big((x-t)y_{10}(t) + (x-t)y_{20}(t)\big)dt\right]$$

$$= -\frac{1}{907200}x^{10} + \frac{1}{20160}x^8 + \frac{1}{2520}x^7$$

$$y_{22}(x) = I_x^{'}\left[\int_0^x \left((x-t)(y_{10}(t) + y_{11}(t)) - (x-t)(y_{20}(t) + y_{21}(t))\right)dt\right] - I_x^{'}\left[\int_0^x \left((x-t)y_{10}(t) - (x-t)y_{20}(t)\right)dt\right]$$
$$= -\frac{1}{9979200}x^{11} - \frac{1}{20160}x^{8} + \frac{1}{2520}x^{7}$$

$$y_{13}(x) = -\frac{1}{21794572800}x^{14} - \frac{1}{1556755200}x^{13} + \frac{1}{907200}x^{10}$$

$$y_{23}(x) = \frac{1}{21794572800}x^{14} - \frac{1}{1556755200}x^{13} + \frac{1}{9979200}x^{11}$$

$$y_{14}(x) = -\frac{1}{2615348736000}x^{16} + \frac{1}{21794572800}x^{14} + \frac{1}{1556755200}x^{13}.$$

$$y_{24}(x) = -\frac{1}{44460928512000}x^{17} - \frac{1}{21794572800}x^{14} + \frac{1}{1556755200}x^{13}.$$

and so on, in the same manner, the rest components can be obtained. The sum of the first five terms is:

$$y_1(x) = x + \frac{1}{2}x^2 - \frac{1}{2615348736000}x^{16}$$

$$y_2(x) = x - \frac{1}{2}x^2 - \frac{1}{44460928512000}x^{17}$$

It is obvious that the iterations converge to the exact solutions $y_1(x) = x + \frac{1}{2}x^2$ and $y_2(x) = x - \frac{1}{2}x^2$ as the number of iterations becomes large.

In Figure 1, we have plotted $y_1(x) = \sum_{m=0}^{4} y_{1m}(x)$ and $y_2(x) = \sum_{m=0}^{4} y_{2m}(x)$.

Example 2

Consider the system of second-order linear integro-differential equations

$$y_1''(x) = -x^3 + x^4 + \int_0^x \left(3y_2(t) + 4y_3(t)\right)dt,$$
$$y_2''(x) = 2 + x^2 - x^4 + \int_0^x \left(4y_3(t) - 2y_1(t)\right)dt, \quad (15)$$
$$y_3''(x) = 6x - x^2 + x^3 + \int_0^x \left(2y_1(t) - 3y_2(t)\right)dt$$

with the initial conditions:

$y_1(0) = 0,\ y_1'(0) = 1.$

$y_2(0) = 0,\ y_2'(0) = 0.$

$y_3(0) = 0,\ y_3'(0) = 0.$

The system of integro-differential equation is equivalent to the system of integral equation:

$$y_{10}(x) = x - \frac{1}{20}x^5 - \frac{1}{30}x^6 + I_x^2\left[\int_0^x \left(3y_2(t) + 4y_3(t)\right)dt\right]$$
$$y_{20}(x) = x^2 + \frac{1}{12}x^4 - \frac{1}{30}x^6 + I_x^2\left[\int_0^x \left(4y_3(t) - 2y_1(t)\right)dt\right]$$
$$y_{30}(x) = x^3 - \frac{1}{12}x^4 + \frac{1}{20}x^5 + I_x^2\left[\int_0^x \left(2y_1(t) - 3y_2(t)\right)dt\right]$$

Let $N_1(y) = I_x^2\left[\int_0^x \left(3y_2(t) + 4y_3(t)\right)dt\right]$,

$N_2(y) = I_x^2\left[\int_0^x \left(4y_3(t) - 2y_1(t)\right)dt\right]$ and

$N_3(y) = I_x^2\left[\int_0^x \left(2y_1(t) - 3y_2(t)\right)dt\right]$

we obtain the first few components of the new iterative method solution.

The first four terms are:

$$y_{10}(x) = x - \frac{1}{20}x^5 - \frac{1}{30}x^6$$
$$y_{20}(x) = x^2 + \frac{1}{12}x^4 - \frac{1}{30}x^6$$
$$y_{30}(x) = x^3 - \frac{1}{12}x^4 + \frac{1}{20}x^5$$
$$y_{11}(x) = \frac{1}{20}x^5 - \frac{1}{2520}x^7 - \frac{1}{5040}x^9 + \frac{1}{30}x^6 + \frac{1}{1680}x^8$$
$$y_{21}(x) = \frac{1}{30}x^6 - \frac{1}{630}x^7 + \frac{1}{1120}x^8 - \frac{1}{12}x^4 + \frac{1}{7560}x^9$$
$$y_{31}(x) = \frac{1}{12}x^4 - \frac{1}{3360}x^8 + \frac{1}{15120}x^9 - \frac{1}{20}x^5 - \frac{1}{840}x^7$$

$$y_{12}(x) = -\frac{1}{75600}x^{10} + \frac{1}{665280}x^{11} + \frac{1}{1995840}x^{12} + \frac{1}{2520}x^7 + \frac{1}{5040}x^9 - \frac{1}{1680}x^8$$

$$y_{22}(x) = -\frac{1}{415800}x^{11} + \frac{1}{1995840}x^{12} - \frac{1}{181440}x^{10} + \frac{1}{630}x^7 - \frac{1}{1120}x^8 - \frac{1}{7560}x^9$$

$$y_{32}(x) = \frac{1}{181440}x^{10} - \frac{1}{1663200}x^{12} + \frac{1}{665280}x^{11} + \frac{1}{3360}x^8 - \frac{1}{15120}x^9 + \frac{1}{840}x^7$$

$$y_{13}(x) = -\frac{1}{165110400}x^{14} - \frac{1}{3027024000}x^{15} + \frac{1}{311351040}x^{13} + \frac{1}{75600}x^{10} - \frac{1}{1995840}x^{12}$$

$$y_{23}(x) = \frac{1}{35380800}x^{13} - \frac{17}{1362180000}x^{15} - \frac{1}{242161920}x^{14} + \frac{1}{415800}x^{11} - \frac{1}{1995840}x^{12}$$

$$y_{33}(x) = -\frac{1}{172972800}x^{13} + \frac{17}{3632428800}x^{14} - \frac{1}{54486432200}x^{15} - \frac{1}{181440}x^{10} + \frac{1}{1663200}x^{12}$$

and so on, in the same manner, the rest components can be obtained.

The sum of the first four terms is:

$$y_1(x) = x - \frac{1}{165110400}x^{14} - \frac{1}{3027024000}x^{15} + \frac{1}{311351040}x^{13}$$

$$y_2(x) = x^2 + \frac{1}{35380800}x^{13} - \frac{17}{13621608000}x^{15} - \frac{1}{242161920}x^{14}$$

$$y_3(x) = x^3 - \frac{1}{172972800}x^{13} + \frac{17}{13621608000}x^{14} - \frac{1}{5448643200}x^{15}.$$

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Example 3

Consider the system of nonlinear third-order Volterra integro-differential equation:

$$y_1'''(x) = -2x - 2x^3 - \frac{2}{5}x^5 + \int_0^x \left(y_1^{\,2}(t) + y_2^{\,2}(t)\right)dt, \quad (16)$$

$$y_2'''(x) = -\frac{2}{3}x^3 - -\frac{1}{5}x^5 + \int_0^x (x-t)\left(y_1^{\,2}(t) - y_2^{\,2}(t)\right)dt,$$

with the initial conditions:

$y_1(0) = 1,\ y_1'(0) = 1, y_1''(0) = 2$

$y_2(0) = 1,\ -y_2'(0) = 1,\ y_2''(0) = 2$

As the above examples, from (3.16), we obtain:

The system of integro-differential equation is equivalent to the system of integral equation:

$$y_1(x) = 1 + x + x^2 - \frac{1}{12}x^4 - \frac{1}{60}x^6 - \frac{1}{840}x^8 + I_x^3\left[\int_0^x \left(y_1{}^2(t) + y_2{}^2(t)\right)dt\right]$$

$$y_2(x) = 1 - x + x^2 - \frac{1}{180}x^6 - \frac{1}{1680}x^8$$

$$+ I_x^3\left[\int_0^x (x-t)\left(y_1{}^2(t) - y_2{}^2(t)\right)dt\right]$$

Let

$$N_1(y) = I_x^3\left[\int_0^x \left(y_1{}^2(t) + y_2{}^2(t)\right)dt\right],$$

$$N_2(y) = I_x^3\left[\int_0^x (x-t)\left(y_1{}^2(t) - y_2{}^2(t)\right)dt\right]$$

$$y_{10}(x) = 1 + x + x^2 - \frac{1}{12}x^4 - \frac{1}{60}x^6 - \frac{1}{840}x^8$$

$$y_{20}(x) = 1 - x + x^2 - \frac{1}{180}x^6 - \frac{1}{1680}x^8$$

$$y_{11}(x) = I_x^3\left[\int_0^x \left(y_{10}{}^2(t) + y_{20}{}^2(t)\right)dt\right]$$

$$= \frac{1}{12}x^4 + \frac{1}{6567734400}x^{20} + \frac{1}{1586304000}x^{18} + \frac{23}{1981324800}x^{16} - \cdots$$

$$y_{21}(x) = I_x^3\left[\int_0^x (x-t)\left(y_{10}{}^2(t) - y_{20}{}^2(t)\right)dt\right]$$

$$= \frac{1}{22973220704000}x^{21} + \frac{1}{42195686400}x^{10} + \frac{1}{16841260800}x^8 + \cdots$$

$$y_{12}(x) = \frac{1}{10080}x^8 + \frac{31601}{1596591942865920000}x^{22} - \frac{1}{15252723936570250690560000000}x^{39} - \cdots$$

$$y_{22}(x) = \frac{134179}{17562511371525120000}x^{22}$$

$$+ \frac{112417}{7164564811401530886145228000000}x^{42} + \cdots$$

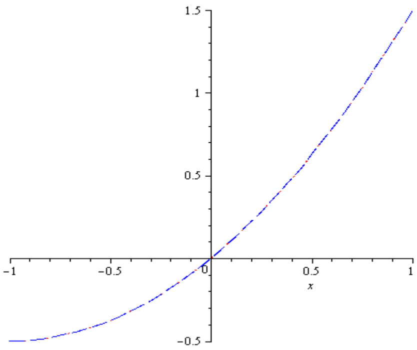
$$\cdots$$

and so on, in the same manner, the rest components can be obtained.

The sum of the first three terms is:

$$y_1(x) = 1 + x + x^2 - \frac{31601}{159659194286592 00}x^{22} + \cdots$$

$$y_2(x) = 1 - x + x^2 + \frac{134179}{1756251137152512 0000}x^{22} + \cdots$$



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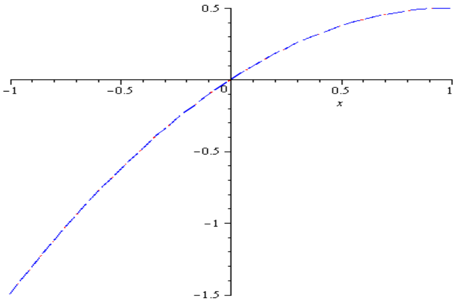
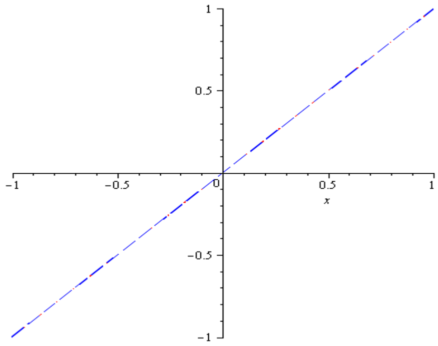


(b)

Figure 1: Exact and approximate solutions for (a) $y_1(x)$ and (b) $y_2(x)$ of Eq. (14), where the red and the blue represent the approximate and exact solutions respectively.



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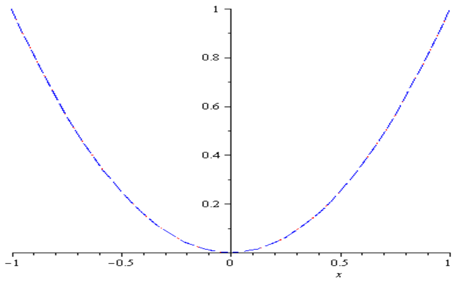

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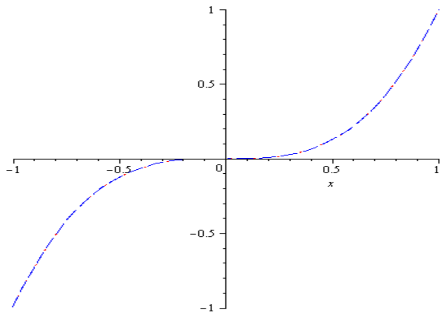

(c)

Figure 2: Exact and approximate solution for (a) $y_1(x)$, (b) $y_2(x)$ and (c) $y_3(x)$ of Eq. (4.5), where red and blue represent the approximate and exact solutions respectively.

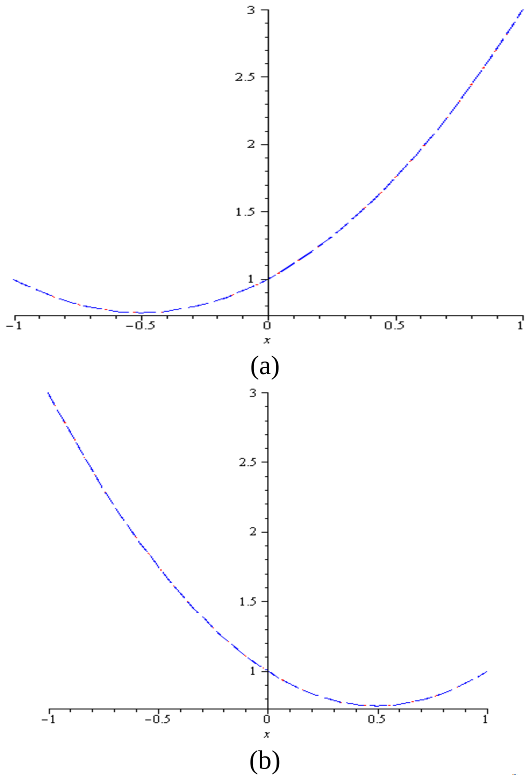

Figure 3: Exact and approximate solutions for (a) $y_1(x)$ and (b) $y_2(x)$ for Eq. (16), where the red and blue represent the approximate and exact solution respectively.

5. Conclusion

In this paper, we successfully applied the new iterative method to find the solution of system of the nth-order linear and nonlinear Volterra integro-differential equations. The present method converts a system of Volterra integro-differential equation to a system of Volterra integral equation. It is clear from the graphs that the solutions agree well with the exact solutions for these equations. The results showed that the method is very accurate and simple.

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