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An Investigation of Buoyancy Driven Natural Convection in a Cylindrical Enclosure

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Abstract: ***An investigation of heat transfer was done in a cylindrical enclosure. The effects of varying Reynolds Number, Prandtl number, Froude number, and Euler number on the temperature and velocity was investigated. In the study, the top surface of a vertical cylindrical container was cooled, the bottom surface heated and the vertical cylindrical walls were considered as being adiabatic. The non-linear governing equations subject to Boussinesq approximation were solved using finite difference method. Results were analysed and presented in graphs to understand the fluid flow and temperature profiles at various locations in the enclosure. Results indicated that buoyancy forces due to temperature difference between the bottom and top of the cylinder are important in determining the velocity of air in the enclosure.***

Keywords: buoyancy force, convective heat transfer, heat transfer, turbulent flow

1. Introduction

Many buoyancy-induced flows are unstable. This instability results from the tendency to motion present in the temperature-stratified medium in which denser fluid overlies the less dense fluid. This denser fluid sitting on top of a less dense fluid is inherently unstable and any disturbance tumbles the fluid. Now once heating is done a disturbance is created, then the denser fluid falls and less dense raises creating vortices. Then the fluid adjacent to the hotter surface rises and that adjacent to the cooler ones falls, setting off a reactionary motion within the enclosure that enhances heat transfer through the enclosure. We encounter many transport processes in fluids in which the motion is driven by the interaction of density differences.

Our focus was on turbulent flow where the fluid particles move in very irregular paths causing an exchange of momentum from one portion of the fluid to the next. The turbulent mixing currents aid the heat transfer. The temperature differences occur in the boundary layer region near the surface and this differences cause density gradients in the ambient medium. In the current study, the heat transfer behaviour in a cylinder containing pure air was studied numerically in order to determine the cooling rate to be applied to cans of beverages and storage vessels for fluids. Also, the results will provide a fast and inexpensive method for the prediction of centre and axial temperature.

The objectives of this study were:
1) To study temperature and velocity profiles in the enclosure.
2) To study effects of varying Re, Pr, Fr and Eu numbers on temperature and velocity profiles.

2. Literature Review

Buoyancy driven natural convection in an enclosure has been studied extensively. Some of the related studies are discussed below;

Lemembre A. and Petit J.P [1] did studies on laminar natural convection in a laterally heated and upper cooled vertical cylindrical enclosure. The influence of the characteristic parameters of the problem on the steady state solution was analysed that is Rayleigh and Prandtl numbers. They discovered that stationary convective heat transfer is more important at the top surface and is explicitly independent of the prandtl Number. A three dimensional unsteady numerical computation study was done by Cheng *et al* [2] to investigate the effects of the thermal boundary condition on the convection flow in a vertical, bottom heated cylinder containing air. Results indicated that the buoyancy driven flow with the sidewall perfectly conducting remains steady for a much wider range of Ra. Another numerical investigation was done by Rahman *et al* [3] to analyse the steady flow and thermal fields with an in-built heat conducting solid circular obstruction. Results indicated that as the Re and Pr numbers increase heat transfer rate increases but temperature at the cylinder centre decreases.

Sigey *et al* [4] investigated on buoyancy driven free convection turbulent heat transfer in an enclosure. A 3-D enclosure in form of a rectangular enclosure containing a convectional heater built into one wall and having a window in the same wall was studied. Results indicated that the enclosure is stratified into three regions: cold upper region hot region in the area between the heater and the window and a warm lower region. Another experimental study was done by Totala *et al* [5] to study natural convection phenomenon from a vertical cylinder. The average heat transfer coefficient along the length of the cylinder was determined. Results indicated that heat transfer coefficient has a maximum value at the beginning due to the development of the boundary layer and decreases in the upward direction due to thickening of boundary layer. Literature review clearly indicated that flow structures driven by the thermal buoyancy in a vertical cylindrical enclosure remain largely unexplored.

3. Mathematical Formulation

Turbulence is widely existed in nature and industrial flows usually occurs at large Reynolds numbers, therefore, turbulence prediction is essential in many engineering designs that involve the calculation of heat and mass transfer. A cylindrical enclosure was considered and the heater placed at the centre as shown below.

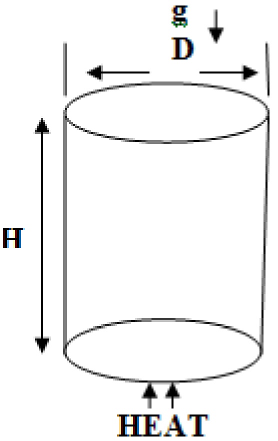

Figure 1: Geometry of the study.

4. Governing Equations

The governing equations of continuity, momentum and energy were used. To avoid the numerical difficulties associated with cylindrical coordinate system, Cartesian coordinate system was used.

4.1 Continuity equation

According to Currie [6]

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u_j)}{\partial x_j} = 0 \qquad (1)$$

For steady state, equation (1) becomes

$$\frac{\partial(\rho u_j)}{\partial x_j} = 0 \qquad (2)$$

4.2 Momentum Equation

$$\frac{\partial}{\partial x_i}(\rho u_i) + \frac{\partial}{\partial x_j}(\partial u_j u_i) = \rho F_i + \frac{\partial}{\partial x_j}(\Pi_{ij}) \qquad (3)$$

The fluid under consideration is Newtonian in nature the tensor is decomposed as

$$\frac{\partial}{\partial x_i}(\rho u_i) + \frac{\partial}{\partial x_j}(\rho u_j\, u_i) = -\frac{\partial \rho}{\partial x_i} + \rho g_i + \frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) + \mu_s\, \delta_{ij}\frac{\partial u_k}{\partial x_k}\right] \qquad (4)$$

4.3 Energy Equation

$$\rho\frac{\partial h}{\partial t} + \frac{\partial}{\partial x_j}(\rho u_j h) = \frac{\partial p}{\partial t} + \frac{\partial}{\partial x_j}(u_j p) - \frac{\partial q_j}{\partial x_j} + \Phi \qquad (5)$$

Where $\Phi = \tau_{ij}\frac{\partial u}{\partial x_j}$

$$\frac{\partial}{\partial t}(\rho c_p T) + \frac{\partial}{\partial x_j}(\rho\, c_p\, u_j\, T) = \frac{\partial}{\partial x_j}\left(\lambda\, \frac{\partial T}{\partial x_j}\right) + \beta T\left(\frac{\partial p}{\partial t} + \frac{\partial u_j p}{\partial x_j}\right) + \phi \qquad (6)$$

The equations together with boundary conditions were used to determine the velocity component and temperature which are the fluid properties.

5. Method of Solution

5.1 Boussinesq approximation

Acccording to Boussinesq [7] the equations governing natural convection within the cylinder are presented in non dimensional form as

$$\frac{\partial U_j}{\partial x_j} = 0 \qquad (7)$$

The momentum equation becomes

$$\frac{\partial u_j}{\partial t} + u_i\frac{\partial u_j}{\partial x_j} + u_j\frac{\partial u_i}{\partial x_j} = -\frac{M_1}{\rho_R}\frac{\partial p}{\partial x_j} - M_2\,\Theta g_1 + M_3\frac{\partial^2 u_i}{\partial x_j^2} + M_3\frac{\partial^2 u_j}{\partial x_j\,\partial x_i} - \frac{\partial}{\partial x_j}\left(\frac{2}{3}\,k\partial_{ij} - v_t\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right)\right) \qquad (8)$$

This simplifies to

$$\frac{\partial u_j}{\partial t} + u_j\frac{\partial u_i}{\partial x_j} = -\frac{M_1}{\rho_R}\frac{\partial p}{\partial x_j} - M_2\,\Theta g_1 + M_3\frac{\partial^2 u_i}{\partial x_j^2} \qquad (9)$$

And the energy equation

$$\frac{\partial \Theta}{\partial t} + 2U_j\,\frac{\partial \Theta}{\partial x_j} = T_2\frac{\partial^2 \Theta}{\partial x_j^2}$$

Where $M_1 = \frac{P_R}{\rho_R U_*^2}$ $M_2 = \frac{gLR}{U_\lambda 2}$ $M_3 = \frac{\mu_R}{\rho_R U_* L_R}$

$$T_1 = \frac{P_R}{C_{PR}\rho_{R\Delta} T_*}\; T_2 = \frac{\lambda_R}{C_{PR}\rho_R U_* L_R}\; T_3 = \frac{\mu_R U_*}{C_{PR}\rho_R \Delta T_* L_R} \qquad (10)$$

5.2 Descritization

The partial differential equations were replaced by discrete approximations. The temperature and velocity profile are approximated by values at discrete points, this gave a computational mesh. This was used to solve the dimensionless equations of momentum and energy.

$$\frac{\partial u}{\partial t} = \frac{1}{Re}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - v\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}\right) - \frac{Eu}{Pr}\frac{\partial p}{\partial y} - \frac{gi}{(Fr)^2} \qquad (11)$$

This equation was descritized using hybrid scheme as

$$\frac{U_{i,j+1} - U_{i,j-1}}{k} = \frac{1}{Reh^2}\left\{\left(U_{i-1,j} - 2\,U_{i,j} + u_{i+1,j}\right) + \left(u_{i,j-1} - 2u_{i,j} + u_{i,j+1}\right)\right\} - \frac{v}{2h}\left\{\left(u_{i+1,j} - u_{i-1,j}\right) + \left(u_{i,j+1} - u_{i,j-1}\right)\right\} - \frac{Eu}{Pr} - \frac{gi}{(Fr)^2} \qquad (12)$$

The energy equation in dimensionless form is written as

$$\frac{\partial \Theta}{\partial t} = \frac{1}{PrRe}\left(\frac{\partial^2 \Theta}{\partial x^2} + \frac{\partial^2 \Theta}{\partial y^2}\right) - 2v\left(\frac{\partial \Theta}{\partial x} + \frac{\partial \Theta}{\partial y}\right) \qquad (13)$$

Descritizing using the hybrid scheme

$$\frac{\Theta_{i,j+1}-\Theta_{i,j-1}}{k}=\frac{1}{PrReh^2}\{(\Theta_{i-1,j}-2\Theta_{i,j}+\Theta_{i+1,j})+(\Theta_{i,j+1}-2\Theta_{i,j}+\Theta_{i,j-1})\}-\frac{v}{h}\{(\Theta_{i+1,j}-\Theta_{i-1,j})+(\Theta_{i,j+1}-\Theta_{i,j-1})\} \quad (14)$$

Where Re = 5,500, Pr = 0.71, Eu = 2.71828, Fr = 0.01

6. Results and Discussion

6.1 Temperature graphs

Once the fluid is heated in the cylindrical enclosure a disturbance is created, heat transfer will initially take place by conduction. This process will continue until the point where buoyancy forces within the conduction region overcome the viscous forces and the fluid motion starts. The air at the top of the cylinder will be cooler and denser with a high pressure due to its greater mass while that at the bottom of the cylinder will be less dense. The resulting temperature differences cause the density gradients which strengthens the buoyancy forces. This buoyancy force makes the fluid to rise along the central axis and sink down along the cold lateral side wall and heat exchange continues. At the top of the cylinder the temperatures are comparatively lower due to increased distance from the heat source as shown in fig 2 and 3. As the distance increases from the heat source, there is a temperature decrease after the volume of fluid has passed through the boundary layer which causes a decrease in heat transfer rate and decay begins as shown from fig. 2 and 3.When varying the Reynolds number , the temperature decreases with decrease in Re number also temperature decreases as the height of the cylinder increases. This is because as we move further from the heat source the buoyancy forces which were responsible for fluid motion becomes weak and the viscous forces dominate which retards the fluid motion as shown from fig. 2. The viscous forces tend to resist fluid motion and this viscosity increases as temperature decreases. From fig 3, the temperature is seen to decrease with decrease in the Pr number as the height of the cylinder increases. Increase in Prandtl number implies thinning of thermal boundary layer, therefore the temperature gradient increases with Prandtl.

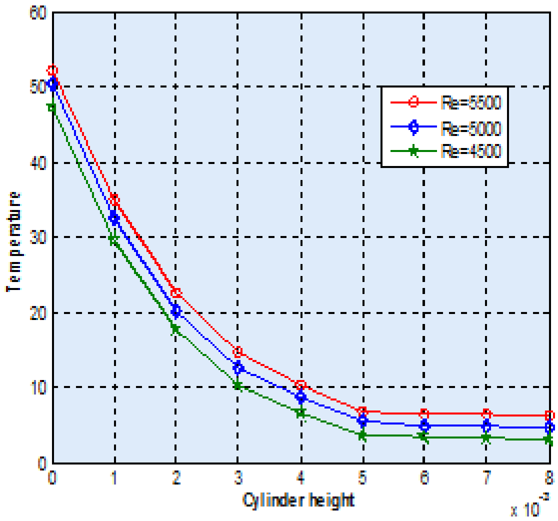

Figure 2: Temperature against height when varying Re. Number.

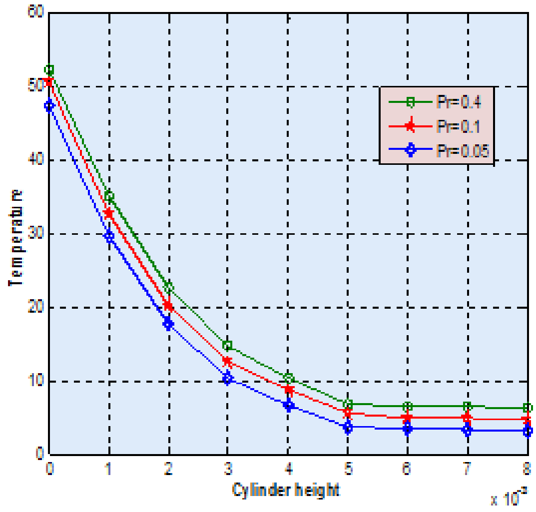

Figure 3: Temperature against height when varying Pr Number.

6.2 Velocity Profiles

The lower part of the cylinder has higher velocities and the upper portion have lower velocities from figs 4,5 and 6.This is because the buoyancy forces created by the density differences are high near the heat source and at the top of the cylinder the buoyancy forces are weak hence the low motion of the fluid particles.

When varying Re number, the velocity is seen to decrease with decrease in Re number as evidenced by fig 4. When the Re number is large the viscous damping action become comparatively less and the fluid velocity increases. But as the Re number decreases the temperature also decreases and the viscous forces dominate and this retards the fluid motion as shown in fig.3. The viscosity increases as temperature decreases.

When we vary the Fr number the velocity decreases with increase in cylinder height. This can be deduced from fig 4. Fr focuses on resistance to flow caused by gravitational effects. As we move further from the heat source the gravitational forces dominate over the buoyancy forces reducing the velocity as deduced from fig 6.

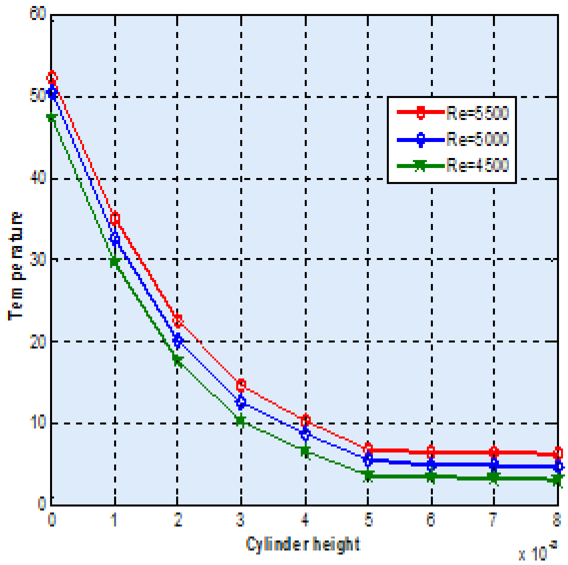

Figure 4: Velocity against height when varying the Re.

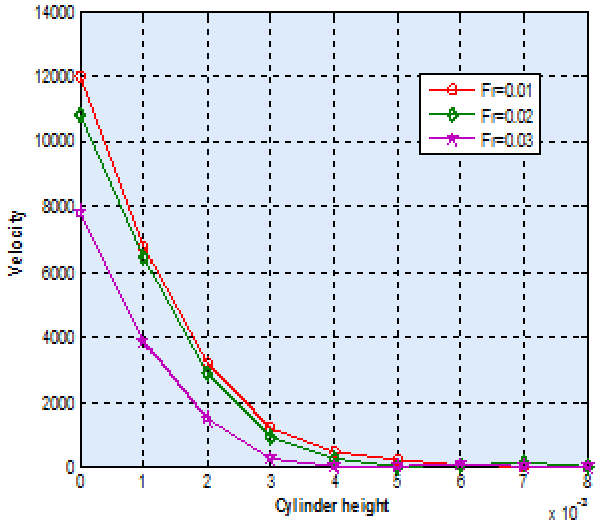

Figure 5: Velocity against height when varying the Fr number.

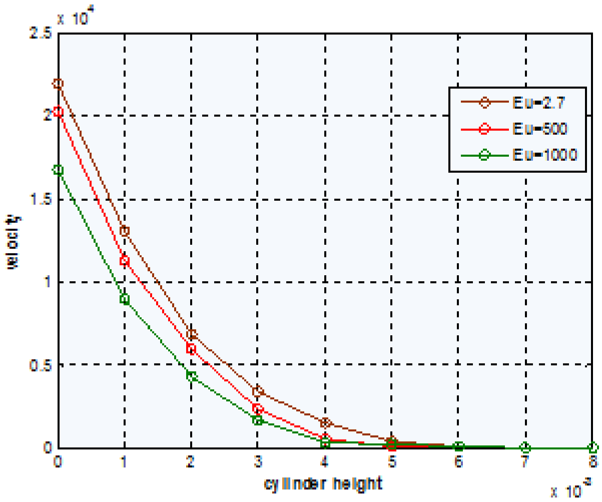

Figure 6: velocity against height when varying the Eu number.

7. Conclusion

An investigation of heat transfer and free convective motion was done in a cylindrical enclosure. The effect of varying the Reynolds, Froude, Euler and Prandtl numbers were investigated. The results indicated that buoyancy forces due to temperature difference between the bottom and top of the cylinder are important in determining the velocity of air in the enclosure. The temperature and velocity decreases with increase in cylindrical height. The study is important as it gives a basis for engineering estimates of temperature differentials, velocities and heat fluxes.

8. Recommendation

1) There is need to investigate flow fields and temperature distribution at various positions of the heater.
2) Studies to be done using cylindrical coordinate system.

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