Zhang's Second Order Traffic Flow Model and Its Application to the Kisii-Kisumu Highway within Kisii County

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Abstract: ***Most highways experience traffic jams as a result of various reasons and hence need for a practical solution. Traffic flow models play an important role in both today's traffic research and in many traffic applications such as traffic flow prediction, incident detection and traffic control. In this project report, I considered the macroscopic Zhang's second order traffic flow model. Macroscopic modeling approach is most suitable for a correct description of traffic flow. Macroscopic traffic flow variables like traffic density and average traffic velocity, which reflect the average state of the road traffic, were considered. It is the variables that define the order of any traffic flow model. The Zhang's second-order macroscopic traffic flow model was applied to the Kisii-Kisumu highway within Kisii County. The viscous continuum traffic flow model was studied numerically using the finite difference approximation method. The flow problem was solved to show that it was stable and efficient. The graphical presentation of numerical results produced verified some well understood qualitative behavior of traffic flow. This model was tested using the Kisii-Kisumu highway and may be extended to other roads.***

Keywords: Macroscopic flow, Traffic Flow, Velocity, Zhang, Traffic density.

1. Introduction

Traffic congestion on motorways is becoming a pressing problem all over the worlds growing economies, Kenya being one of them. This is as a result of rapidly increasing population and the crowding of motorized traffic onto limited road networks. It can also be attributed to traffic breakdown in initially free flowing traffic. On the Kisii-Kisumu road for instance, apart from vehicles; motorcycles have become a major concern. Their sudden upsurge on the road has made smooth flow of traffic almost impossible. Nowadays traffic flow and traffic density has become a major problem in the society. Traffic jams do not only cause considerable costs due to unproductive time losses; they also augment the probability of accidents and have a negative impact on the environment (air pollution, fuel lost, health problems, noise, stress). One-approach to-ease congestion is to increase the capacity of existing roadways by addition of lanes Kimathi (2012). This approach is long term, very costly and often not feasible due to environmental and /or societal constraints. Another approach is metering the rate at which vehicles enter a road network, Gonzales et al (2009). This results in need for a short-term solution that involves controlling traffic in such a way that congestion is solved, reduced or at least delayed.

Traffic flow models have been developed to address this problem. They can be used to simulate traffic, for instance, to evaluate the use of a new part of the infrastructure. Research on the subject of traffic flow modeling started when Lighthill and Whitham (1955) presented a model;

$$\rho_t + (\rho v)_x = 0 \qquad (1.1)$$

The aim of traffic flow analysis is to create and implement a model which would enable vehicles to reach their destination in the shortest possible time using the maximum roadway capacity.

2. Literature Review

Different types of traffic flows are described by different models. For equilibrium link flow, the celebrated Lighthill-Witham-Richards (LWR) model was developed in 1956. They separately developed the first dynamics traffic flow model. The LWR model describes traffic using a conservation law where they assumed that the traffic flow is related to the traffic density.

The LWR model is a first-order model in the sense of a PDE system order. Newell (1993) improved the LWR model so as to cope with Shockwaves and stop-and-go traffic in congested traffic situations. Payne (1971) proposed the first continuum traffic flow model. The macroscopic Payne model is of second order since it has two variables: traffic density and average traffic velocity.

Macroscopic flow variables, such as flow, density, speed and speed variance, reflect the average state of the traffic flow in contrast to the microscopic traffic flow variables, which focus on individual drivers.

Helbing (1997) proposed a third-order macroscopic traffic flow model with the-traffic density, the average velocity and the variance on the velocity as variables. A great deal of work has been devoted to the study of traffic flow by improvement of already existing models through using various numerical approximation techniques in an attempt to give more accurate results Hoogendoorn and Bovy (2006).The manuscript by Lighthill and Whitham (1955) set the tone for many researchers' Investigations into the theory of traffic flow especially for traffic flows on a single, long, and rather idealized road. Verification of traffic flow models has also been done by Wu and Brilon (1999), where they verified the microscopic Cellular-Automaton model on German and American motorways. Moreover, Yonnel et al. (2002) used the microscopic based Paramics traffic simulation model to investigate freeway operation on 1-680 freeways in the San Francisco bay area. The aim was to obtain in-depth knowledge of the Paramics model for freeway applications. Kotsialos et al. (1999) showed that macroscopic traffic flow models are suitable for large-scale simulation of traffic flow in networks.

A new higher-order continuum model has been developed by Zhan'g (2000).

$$\rho_t + (\rho v)_x = 0 \tag{1.2}$$

$$v_t + [v + 2\beta c(\rho)]_{v_x} + \frac{c^2(\rho)}{\rho}\rho_x = \frac{v_*(\rho) - v}{\tau} + \mu(\rho) v_{xx} \tag{1.3}$$

Where

$$\mu(\rho) = 2\beta\tau c^2(\rho) \tag{1.4}$$

$$c(\rho) = \rho v_*'(\rho) \tag{1.5}$$

$\frac{c^2(\rho)}{\rho}\rho_x$ is the anticipation term that describes the response of macroscopic driver to traffic density i.e the space concentration and pressure.

In Kenya, studies have been carried out on traffic flow models on various roads. An example is the Zhang's second order traffic flow model and its application to the Nairobi-Thika highway. Sigey and Kimathi (2006). They analyzed the wave phenomenon of the highway traffic flow during rush hours. A 3- phase traffic theory has also been developed to explain traffic breakdown and the resulting spatiotemporal features of congested vehicular traffic, Kimathi (2012). In this study, the Zhang's second-order macroscopic traffic flow model was considered to investigate the traffic flow phenomena during the rush hours on the Kisii-Kisumu highway in Kisii County.

2.1 Classification of traffic flow models

Traffic models can be classified according to the level and detail traffic flow. Models may be categorized using various dimensions (deterministic or stochastic, continuous or discrete, analytical or simulation etc). The most common classification is the distinction between microscopic and macroscopic traffic flow modelling approaches. However, this distinction is not unambiguous, owing to the existence of hybrid models. This is why models are categorized based on either representation where traffic is either macroscopic or microscopic. The observed behaviour of drivers, that is, headways, driving speeds and driving lane, is influenced by different factors, which can be related to the driver–vehicle combination (vehicle characteristics, driver experience, age, gender and so forth), the traffic conditions (average speeds, densities), infrastructure Conditions (road conditions) and external situational influences (weather, driving regulations).These models are based on the analogy between compressible flow in a Navier-Stokes fluid and traffic flow equations Maria R et al (2012) Over the years, different theories have been proposed to (dynamically) relate the observed driving behaviour to the Parameters describing these conditions. Analysts approach the problem in many main ways, corresponding to the main scales of observation in physics.

2.1.1 Macroscopic traffic flow models

A macroscopic traffic model is a mathematical model that formulates the relationship among traffic flow characteristics like density, flow, mean, speed of traffic system. Such models are conventionally arrived at by integrating microscopic traffic flow models and converting the single entity level characteristics to comparable system level characteristics.

A macroscopic model may assume that the traffic stream is properly allocated to the roadway lanes, and employ an approximation to this end. The method of modeling traffic flow at macroscopic level originated under an assumption that traffic streams as whole are comparable to fluid streams. Vehicular motion is however different from the motion of elementary particles. For instance, vehicular motion is directed towards a single direction. Another difference is that unlike in particles where the influence of propagation is strongly biased in one direction, vehicular interaction is strongly influenced by a vehicle right in front of it.

The vehicle in front is affected rather weakly by the vehicle behind it KIS-SNU (2003).

In this research macroscopic traffic flow model has been studied both theoretically and numerically. The study has considered Zhang's second-order traffic flow model developed by Zhang (1998) and in Zhang (2000) where he developed a finite difference scheme for this model. It has been applied on the Kisii-Kisumu highway in Kisii County. The Zhang model is a macroscopic traffic flow model.

2.2 The fundamental diagram on traffic flow

In understanding the relation between the traffic flow and the traffic density at a given location of the highway, a fundamental diagram is essential. The fundamental diagram (FD) of traffic flow is a basic tool in traffic engineering to understand the flow capacity of a roadway, Shima (2013). The diagram is a macroscopic traffic model that describes a statistical relation between the macroscopic traffic flow variables of flow and density. It provides a graphical depiction of the flow of vehicles along the highway over time. The traffic on the highway is observed and the traffic flow (vehicles/hour) versus the traffic density (vehicles/kilometer/lane) is plotted for a location along the highway. A curve as shown below occurs.

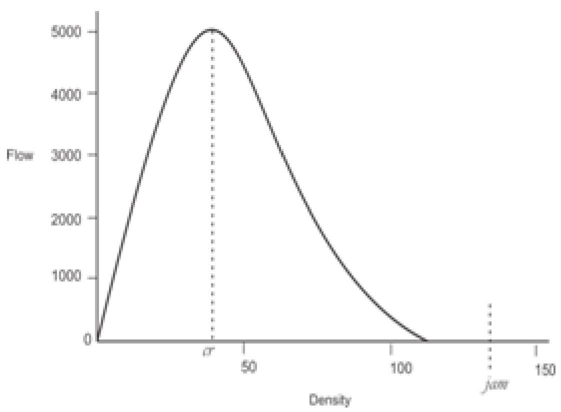

Figure 1: The Fundamental Diagram

3. Methodology

The highway used in this study is a one- way road, with length $0 \leq x \leq L$.

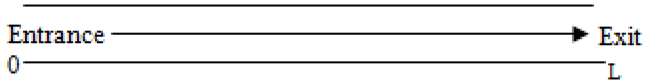

Figure 2.1 A section of the one-way road

The assumption before the study was conducted is that vehicles do not appear or disappear. Therefore the number of vehicles will depend only on the number already present in the system and the flow of the vehicles into and out of the system. The zhang's model is of second-order as it is comprised of two equations in density $\rho(x,t)$ and velocity $v(x,t)$

3.1 The governing equations.

The wave model of traffic flow theory is the simplest dynamic traffic flow model that reproduces the propagation of traffic waves. It is made up of the continuity equation, the conservation of momentum equation and the initial and boundary conditions.

3.1.1The continuity equation.

An important relation in traffic flow theory is the continuity equation. This equation is used to relate the instantaneous characteristic of density to the local characteristic flow, (Zhan'g 1998).

The density measured in car length can vary from 0 on an empty highway to 1 in a bumper to bumper traffic. For a section of a highway with an entry point p_1 and an exit point p_2 we let c_1 be the number of cars passing p_1 in timeΔt and q_1 be the flow.

We also let c_2 be the number of vehicles passing p_2 in the time Δt and q_2 be the flow, (Tom v. 2014).

$p_1 p_2 \longrightarrow$

Then assuming $c_1 > c_2$ then it follows that

$$q_1 = \frac{c_1}{\Delta t}, q_2 = \frac{c_2}{\Delta t}$$

Then

$$\Delta q = q_2 - q_1 = \frac{c_1 - c_2}{\Delta t} = -\frac{\Delta c}{\Delta t}$$

Therefore

$$\Delta c = -\Delta q \Delta t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (a)$$

Similarly for

$$\rho_1 > \rho_2, \Delta\rho = \rho_2 - \rho_1 = \frac{\Delta c}{\Delta x}$$

This implies that

$$\Delta c = \Delta\rho\Delta x \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (b)$$

From (a) and (b) above we get

$$\Delta\rho\Delta x + \Delta q\Delta t = 0$$

Dividing both sides by $\Delta t\Delta x$

We get

$$\frac{\Delta\rho}{\Delta t} + \frac{\Delta q}{\Delta x} = 0$$

This culminates to

$$\frac{\partial\rho}{\partial t} + \frac{\partial q}{\partial x} = 0 \qquad (2.1)$$

It is a mathematical representation of the principle of mass conservation i.e. the amount of flux entering and leaving a given channel/point is the same. A basic conjecture of the simple continuum model is that vehicles are not created or lost along the road.$\rho$is the fluid density and q is fluid flow. Vehicles on the highway are thought as compressible fluids.

3.1.2Conservation of momentum equation.

This equation relates the sum of the forces acting on an element of fluid to its acceleration. It represents the principle of conservation of motion that states that the rate of momentum change is equal to the net forces. The equation of velocity dynamics can be written in the general form as:

$$v_t - v v_x = \frac{V_*(\rho) - v}{\tau} - \frac{c(\rho)}{\rho}\rho_*$$

Where $\tau$the relaxation is time and V_* is equilibrium velocity

$$\frac{V_*(\rho) - V}{\tau}$$

is the relaxation or acceleration term. This term forces vehicle velocity towards equilibrium. $C(\rho)$ is an anticipation term which Payne(1971) suggested should be of the form $C(\rho) = \delta\big(\rho\Theta(\rho)\big)$and Zhan'g (1998) suggesting that the local sound speed be $(\rho) = c_0^2$. When the relaxation term in equation 2.2 is zero, then the Zhang's model reduces to the LWR model. Density was measured in units of vehicle/meter with a maximum of $\rho_{max} = 0.2veh/m$ basing the average vehicle for length at 5m. Velocity was measured in units of meters/second, with a maximum of $30m/s$.He further included a viscosity term to smooth out sudden density and velocity changes deemed unrealistic.

As modified by Zhan'g (1998) the model is governed by the equation:

$$v_t + v v_x = \frac{v_*(\rho) - v}{\tau} - \frac{c^2}{\rho}\rho_x + \mu\frac{v_{xx}}{\rho} \qquad (2.2)$$

Where $\mu$is the viscosity or dissipative constant.

$v v_{x=} [v + 2\beta c(\rho)]_{v_x}$.

$$\mu(\rho) = 2\beta\tau c^2(\rho) \qquad (2.3)$$

$$c(\rho) = \rho v_*'(\rho). \qquad (2.4)$$

Flow of traffic was analysed using equation 2.2 and 2.3.

3.2 Initial and Boundary Conditions

The initial conditions must be defined to solve a problem using this model. A boundary is defined to be$\rho(x,t)$, representing density as a function of time and position. These boundaries typically take two different forms resulting in the initial value problem (IVP) and the boundary value problem (BVP). The initial value problem gives the traffic densities at time t=0, such that $\rho(x,0)$=$\rho_1(x)$ where $\rho_1(x)$ is the given density function similarly velocity at times t=0 is given by $v(x,0) = v_*(\rho_1(x))$

Boundary value problem gives some functions $\hat{\rho}(t)$ such that $\rho(0,t) = \hat{\rho}(t)$

Similarly Velocity is given by $v(0,t) = v_*(\hat{\rho},t)$

The initial conditions are

$$\rho(x,0) = \rho_{\prime}(x) \qquad (2.5)$$

$$v(x,0) = v_*(\rho_{\prime}(x)) \qquad (2.6)$$

And the boundary conditions are

$$\rho(0,t) = \hat{\rho}(t) \quad (2.7)$$
$$v(0,t) = v_*(\hat{\rho}(t)) \quad (2.8)$$
$$\rho_{xx}(L,t) = 0 \quad (2.9)$$
$$v_{xx}(L,t) = 0 \quad (2.10)$$

The initial condition function $\rho_{\prime}(x)$equation 2.5 was set arbitrarily to avoid numerical instabilities. Boundary conditions 2.7 and 2.8 correspond to a specified input $\hat{\rho}$ onto the highway.

It was assumed that input density arrived in equilibrium. At the outgoing boundary, $x = L, \rho_{xx}$ and v_{xx} are set to be zero to allow travelling wave Solution to pass through the boundary without creating any artificial reflections.

The model equations are discretized as:

$$\frac{\rho_i^{k+1} - \rho_i^k}{dt} = \frac{-\rho_i^k v_i^k - \rho_{i-1}^k v_{i-1}^k}{dx} \quad (2.11)$$

And

$$\frac{v_i^{k+1} - v_i^k}{dt} = -[v_i^k + 2\beta c(\rho_i^k)](v_x)_i^k - \frac{c^2(\rho_i^k)}{\rho_i^k}\left[\frac{\rho_i^k - \rho_{i-1}^k}{dx}\right] + \frac{v_*(\rho_i^k) - v_i^k}{\tau} + \mu(\rho_i^k)\left[\frac{v_{i+1}^k - 2v_i^k + v_{i-1}^k}{dx^2}\right] \quad (2.12)$$

The convective wave speed is given by

$$a = (v_i^k + 2\beta c(\rho_i^k)) \quad (2.13)$$

So that

$$(v_x)_i^k = \begin{cases} \dfrac{[v_i^k - v_{i-1}^k]}{dx} & \text{if } a \geq 0 \\ \dfrac{[v_{i+1}^k - v_i^k]}{dx} & \text{if } a < 0 \end{cases} \quad (2.14)$$

Equations 2.9 and 2.10 are discreticized to yield

$$\rho_n^k = \frac{5\rho_{n-1}^k - 4\rho_{n-2}^k + \rho_{n-3}^k}{2} \quad (2.15)$$
$$v_n^k = \frac{5v_{n-1}^k - 4v_{n-2}^k + v_{n-3}^k}{2} \quad (2.16)$$

The next chapter is a summary of the results obtained. The Zhang's second order model is discussed, and its analytical and numerical solutions developed.

4. Results and Discussions

To test the model, a section of the highway was selected. The selected section was 500m Long from DarajaMbili market towards the Kisii-Migori junction and was partitioned as a piece of roadway i.e. an interval (a,b) of four sections i.e$L = 500$m and $n = 4$. At each zone, the average of density and velocity over time were computed.

The initial density profile was given by

$$\rho_{\prime}(x) = 0.3 + 0.1(e^{-0.1(x-100)^2}) \quad (3.1)$$

The input density in this case was

$$\hat{\rho}(t) = 0.3 - 0.2\sin(t) \quad (3.2)$$

This indicates a concentrated mass of high density traffic. The goal of this input density was to identify the basic features encountered when a section of high density traffic or a section of low density traffic enters the roadway. The values of the input parameters are listed in table 3.1.

Table 1: Input Parameters

Parameters	*Value*	*Units*
β	1	dimensionless
τ	0.1	S

The wave solutions are shown in the figures below.

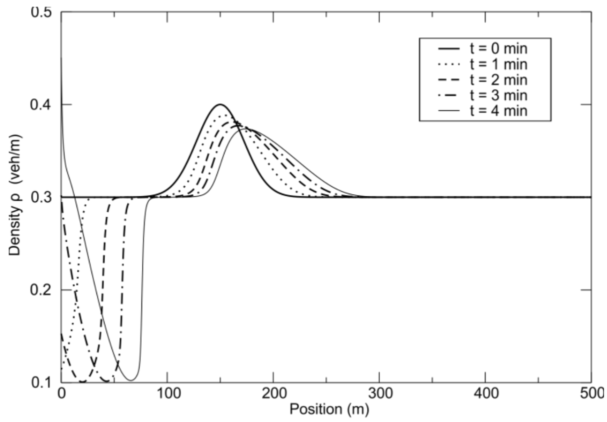

Figure 3.1: Early Time (density)

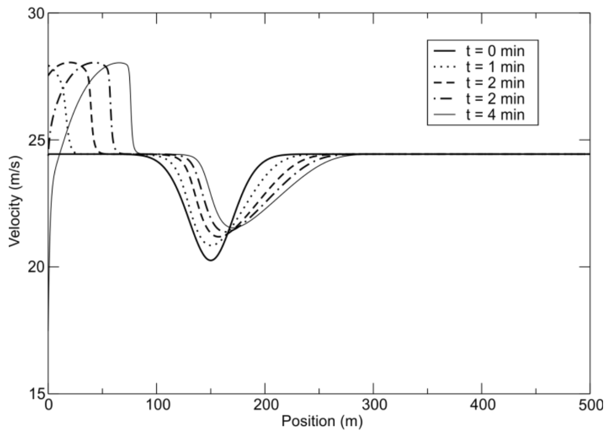

Figure 3.2: Early Time (Velocity)

4.1 Early Time Traffic Behaviour

The early time densities and velocities of the incoming traffic with respect to different positions on the highway were observed. The results are as shown in figures 3.1 and 3.2. The road was divided into four sections which acted as sensor points. i.e. 0m to 100m, 100m to 200m, 200m to 300m and finally 300m to 500m. In the four sections, the observations were made every minute within four minutes. The density and velocity of traffic were observed during the same intervals. At t= 0 min, observations on the initial cluster of traffic reveals a density of 0.3 veh/m, indicating a bumper to bumper scenario. The corresponding velocity is 24m/s. a minute later, the density slightly drops with velocity increasing to 25m/s. The situation remains the same up to 100m position. As the flow approaches 150m, the density increases to 0.4 veh/m, with the velocity dropping to 20m/s. the traffic situation is again bumper to bumper. This can be attributed to the car-following behaviour of the vehicles behind the initial cluster. As the initial cluster begins to spread and as the flow approaches 200m, the traffic ahead sees clearer and increases its speed. Space is created for the vehicles behind this cluster to equally increase their speed. Consequently, the density is now oscillating around 0.35 veh/m. The velocity is between 22m/s and 24m/s. The slight fluctuations in density and velocity at these different times are minimal though. As the flow reaches 300m, the density now completely reduces to 0.3 veh/m with the velocity increasing and converging to 25 m/s. From the 300m position, and with clear vision, the density is maintained at 0.3veh/m with vehicular velocity maintaining at 25m/s up to the 500m position. The input density becomes low and traffic simply propagates for early time.

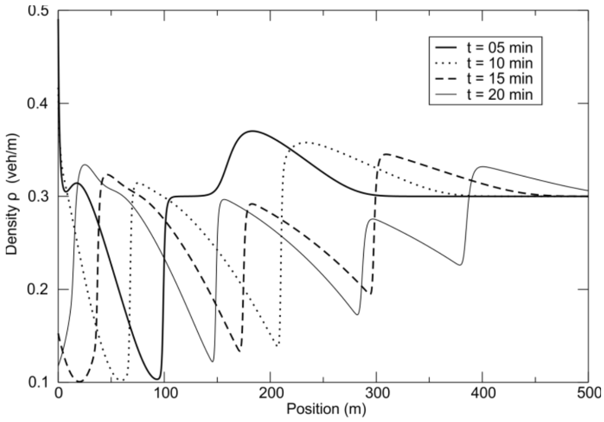

Figure 3.3: Late Time (Density)

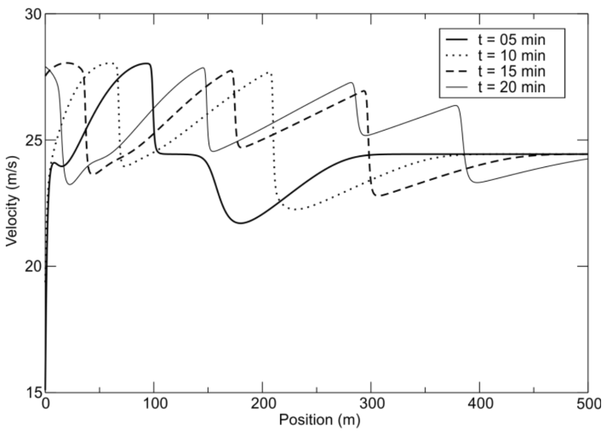

Figure 3.4: Late Time (Velocity)

4.2 Late Time Traffic Behaviour

The late time density and velocity behaviours with respect to different positions on the grid exhibited near similar trends with the observations in early time. The observations are shown in figure 3.3 and 3.4. The observations were made at intervals of 5 minutes as the final cluster of vehicles left the grid the new incoming cluster sees higher density ahead slowing down. Vehicular velocity reduces from 25m/s to 20 m/s. similarly, vehicular density increases from 0.3veh/m to 0.4 veh/m. In the first 100m, the density at t=5min drops from 0.4 to 0. This is attributed to car following behaviour attributed to macroscopic flow phenomenon. i.e., after briefly slowing down and with the vehicles moving at the same velocity, more space was created thereby allowing more spread. This in effect enabled the cluster of cars behind to increase their velocity thereby considerably decreasing density. Vehicular velocity increased from 20m/s to 28m/s. between 100m and 200m, at t=10min, vehicular density once again increases to 0.03veh/m with the velocity slightly dropping to 27m/s. density further drops to 0.2 veh/m15 minutes later, with velocity further dropping to 26m/s.

After 20 minutes, the initial cluster spreads and disperses considerably. As the initial cluster spreads, the bunched input density is allowed to begin to spread and speed up at t=20min.

5. Conclusion

In this study, from a practical point of view, a consideration was made on the possibility of using traffic flow as the state variable for traffic control purpose. A macroscopic traffic flow model within the framework of Zhan'g has been presented. This model was applied on a 500m stretch on the Kisii- Kisumu road. The macroscopic modelling approach to traffic flow applied, reproduced some characteristics of known behaviours of vehicles in a highway. i.e vehicular motion is influenced by a vehicle right in front of it and the vehicle in front is affected rather weakly by the vehicle behind it.

This was clearly the case in this study as shown in both early and late time cases. In both cases, flow rate (veh/unit time), and density (veh/unit distance) had an inverse relationship as shown in figures 3.1 and 3.2 for early time behaviours and figures 3.2 and 3.3 for late time behaviours. The model was studied and implemented numerically using a system of equations i.e the continuity equation and the conservation of momentum equation. The numerical method for solving the macroscopic model in conservative form was discussed and tests carried out to show the effectiveness of the method. It is noted that the flow in highways is dictated by the conditions upstream and downstream. Velocity of the traffic therefore depends on the density on the road.

This report builds a solid base for modelling traffic flow macroscopically. The method can be extremely promising in that results resemble hypothesized real world results.

6. Recommendations

Modelling of traffic flow is becoming increasingly important. However the development of new realistic traffic flow models is still a challenging task. For example there are few models that accurately capture other traffic dynamics like mergers and diversions, abrupt lane changing, driver-vehicular interaction and many more. It is therefore recommended in this study that, besides developing new models, there is need to validate traffic flow models that have been proposed such as the PW model. Another direction of research in computation may come from the development of parallel algorithms to simulate traffic flow in large networks.

Recommendations are also made that in future ramp modelling should be the next logical focus of similar research. Data will be taken from traffic sensors at the input of the sections on the highway to be studied. The method provided in this research can be extended to solve other higher order traffic flow models like the multi-lane traffic simulations

For traffic flow at aggregate level, higher order traffic flow models may be combined in order to capture more dynamics. For example the PW model and the Zhang's model can be combined for link flows. This can address modelling at mergers and divergers.

References

[1] Gonzales, Erick J, Chavis and Daganzo F. Multimodal Transport Modelling for Nairobi Kenya: Insights and Recommendations with evidence-based Model. Working Paper UCB-ITS-VWP-2009-**5**
[2] Hoogendoorn, S.P. and P.H.L. Bovy (2000a). Modelling Multiple User-Class Traffic Flow. Transportation Research **B (34) 2**, 123-146.
[3] HaoXu, Hongchao Liu, Huaxin Gong, (2013), Modeling the asymmetry in traffic flow: A microscopic approach. Texas Technical University U.S.A.
[4] Kerner,B.S. (1999).Theory of Congested Traffic Flow: Self-Organization without Bottlenecks. Proceedings of the 14th International Symposium on Transportation and Traffic Theory, Jerusalem.
[5] KIAS-SNU (2003). A Mathematical Model of Traffic Flow. A research paper for KIAS-SNU Physics Winter Camp.
[6] Kimathi M and Sigey J, (2006), Zhangs second order traffic flow model and its application to the Nairobi-Thika highway in Kenya. An MSC Thesis, JKUAT.
[7] Kimathi E.M (2012), Mathematical Models for 3-Phase Traffic Flow Theory. A PhD thesis, University of Kaiserslautern Germany.
[8] Kotsialos, A., M. Papageorgiou, and A. Messmer, (1999). Optimal Co-ordinate and Integrated Motorway Network Traffic Control, Proceedings of the 14th International Symposium of Transportation and Traffic Theory, Jerusalem **15**pp 621-647
[9] Lighthill, M.H., and G.B. Whitham (1995). On kinematics waves II: a theory of traffic flow on long, crowded roads. Proceedings of the Royal Society of London Series A, pp 229,317-345.
[10] Payne, H. J. (1971). Models for Freeway Traffic and Control. Simulation Councils Proceedings on. Math Models, **28**pp 51-61.
[11] Richards P.I (1956), shockwaves on the highway, operations research **4**pp 42-51.
[12] Shima O, Benjamin L (2011). Modelling the Asymmetry in traffic flow. Texas Technical University U.S.A.
[13] Yonnel Gardes, A.D. May, J. Dahlgren, A. Skarbardonis (2002). Bay Area Simulation and Ramp Metering Study. California PATH Research Report, UCB-ITS- PRR, 7.
[14] Zhang, H.M. (1998).A theory of Non- equilibrium Traffic Flow. Transportation research part B. **32**, pp 485-498.
[15] Zhang, H. M. (2002). A Non-Equilibrium Traffic Model Devoid of Gas-Like Behavior. Transportation Research Part B. **36**pp 275-290.

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