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MHD Slip Flow and Convective Heat Transfer of Nanofluids over a Permeable Stretching Surface

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Abstract: ***MHD slip flow and convective heat transfer of nanofluids over a vertical stretching surface subjected to injection has been analyzed. Two types of nanofluids such as Copper-Water nanofluid and Alumina-Water nanofluid are considered for the present study. The Boundary layer equations of motion and energy which are non-linear partial differential equations are reduced to non-linear ordinary differential equations by means of similarity transformations. The resulting non-linear ordinary differential equations are solved numerically by most efficient Nachtsheim-Swigert shooting iteration technique for satisfaction of asymptotic boundary conditions along with Fourth Order Runge-Kutta Integration Method. Numerical computations are carried out for various values of slip parameter, Injection parameter, Magnetic interaction parameter, and solid volume fraction. Also the values of Skin friction Coefficient and Nusselt number are obtained numerically and are tabulated.***

Keywords: MHD, Nanofluids, Slip Velocity, Injection

1. Introduction

With the advances in computing technology over the past few decades, electronic devices have become faster, smaller and more powerful, and the vital issue is an ever-increasing heat generation rate from these devices. In most cases, the chips are cooled using forced air flow. However, when dealing with a component that contains billions of transistors working at high frequencies, the temperature can reach a critical level where standard cooling methods are not adequate. Nanofluids can be used to cool these devices such as high – power microwave tubes and high power laser diode arrays. Flow and heat transfer characteristics over a stretching sheet have important industrial applications, for instance, in the extrusion of a polymer sheet from a die. In the manufacturing of such sheets, the melt issues from a slit and is subsequently stretched. The rates of stretching and cooling have a significant influence on the quality of the final product with desired characteristics.

The study of magnetic field effects has important applications in physics, chemistry and engineering. Many industrial equipments, such as magnetohydrodynamic generator, pumps, bearings and boundary layer control are affected by the interaction between the electrically conducting fluid and a magnetic field. The works of many researchers have been studied in relation to these applications. Motivated by these applications, a problem of such kind is dealt with in this work.

After the pioneering work of sakiadis [1], a large number of research papers on a stretching sheet have been published by considering various governing parameters with different types of fluids. Most studies have been concerned with constant surface velocity and temperature Tsou et. al.[2], but for many practical applications the surface undergoes stretching and cooling or heating that cause surface velocity and temperature variations as pointed by Crane[3]. Carragher[4] analysed the same problem of Crane to study heat transfer and obtained the Nusselt number for the entire range of Prandtl number (Pr).

In due course of time, many investigators attempted stretching surface problems and its associated heat and mass transfer aspects in more general situations with different assumptions pertaining to the velocity and temperature. In Dandapat and Gupta[5], an exact analytical solution of the non-linear equation governing this self-similar flow which is consistent with the numerical results in Rajagopal et al.[6] is given and the solutions for the temperature for various values of k1 are presented. Later, Cortell [7] extended the work of Dandapat and Gupta[5] to study the heat transfer in an incompressible second-order fluid caused by a stretching sheet with a view to examining the influence of the viscoelastic parameter on heat-transfer characteristics.

Vajravelu and A. Hadjinicolaou [8] studied the heat transfer characteristics in the laminar boundary layer of a viscous fluid over a stretching sheet with viscous dissipation or frictional heating and internal heat generation. Elbashbeshy et al.[9] studied heat transfer in a porous medium over a stretching surface with intentional heat generation and suction/injection.In the study (Martin and Boyd[10]), they have analyzed slip flow and heat transfer at constant wall temperature. Based on their boundary layer theory, non-equilibrium effects will cause a drop in drag on airfoils. Matthews and Hill[11] discussed the effect of replacing the standard no-slip boundary condition with a nonlinear Navier boundary condition for the boundary layer equations. The hydrodynamic flow in the presence of partial slip over a stretching sheet with suction has been studied by Wang [12].

Many researchers have studied the influences of electrically conducting fluids, such as liquid metals, water mixed with a little acid and other ingredients in the presence of a magnetic field on the flow and heat transfer of an incompressible viscous fluid passing over a moving surface or a stretching plate in a quiescent fluid. Pavlov [13] was one of the first pioneers in this field of study. Linear stretching with suction or blowing in the presence of a constant transverse magnetic field was presented by Chakrabarti and Gupta[14] who obtained a very simple exponential solution. This work has been further extended by Chiam[15] to that of a sheet stretching with a power-law velocity and having a variable magnetic field of a special form that results in a similarity solution.

Cortell [16] also studied the MHD flow of power law fluid over a stretching sheet in which case he has dealt numerically with a non-linear differential equation. Anjali Devi and Thiyagarajan [17] studied the steady nonlinear hydromagnetic flow of an incompressible, viscous and electrically conducting fluid with heat transfer over a surface of variable temperature stretching with a power-law velocity in the presence of variable transverse magnetic field. Char [18] studied MHD flow of a viscoelastic fluid over a stretching sheet by considering thermal diffusion in the energy equation.

Dissipation is the process of converting mechanical energy of downward-flowing water into thermal and acoustical energy. Vajravelu and Hadjinicolaou [19] analyzed the heat transfer characteristics over a stretching surface with viscous dissipation in the presence of internal heat generation or absorption. Partha et al. [20] investigated the effect of viscous dissipation on the mixed convection heat transfer from an exponentially stretching surface. Abel et al. [21] presented a mathematical analysis on momentum and heat transfer characteristics in an incompressible electrically conducting viscoelastic boundary layer fluid flow over a linear stretching sheet by taking into account of viscous dissipation and Ohmic dissipation due to presence of transverse magnetic field and electric field.

Convective heat transfer in nanofluids is a topic of major contemporary interest both in sciences and engineering. Choi[22] was the first to introduce the word nanofluid that represent the fluid in which nanoscale particles (diameter less than 50 nm) are suspended in the base fluid. The base fluids used are usually water, ethylene glycol, toluene and oil. The choice of base fluid–particle combination depends on the application for which the nanofluid is intended. Recent research on nanofluid showed that nanoparticles changed the fluid characteristics because thermal conductivity of these particles was higher than convectional fluids. Nanoparticles are of great scientific interest as they are effectively a bridge between bulk materials and atomic or molecular structures. The common nanoparticles that have been used are aluminum, copper, iron and titanium or their oxides.

Various benefits of the application of nanofluids include improved heat transfer, heat transfer system size reduction, minimal clogging, microchannel cooling and miniaturization of the system. Therefore, research is underway to apply nanofluids in environments where higher heat flux is encountered and the convectional fluid is not capable of achieving the desired heat transfer rate. Convective flow in porous media has been widely studied in the recent years due to its wide applications in engineering as post-accidental heat removal in nuclear reactors, solar collectors, drying processes, heat exchangers, geothermal and oil recovery, building construction, etc. They are also used in other electronic applications which use microfluidic applications. Many researchers like Buongiorno [23], Daungthongsuk [24], Trisaksri [25], and Wang [26] have been actively involved in nanofluid research and Das et al. [27] has additionally written a book on nanofluid.

Ghasemi et al. [28] presented the results of a numerical study on natural convection heat transfer in an inclined enclosure filled with a water–CuO nanofluid. They found that the heat transfer rate is maximized at a specific inclination angle depending on Rayleigh number and solid volume fraction. The natural convective boundary layer flows of a nanofluid past a vertical plate have been described by and Kuznestov and Neild [29]. The presence of the nanoparticles in the fluids increases appreciably the effective thermal conductivity of the fluid and consequently enhances the heat transfer characteristics.

Laminar boundary layer flow of nanofluid over a flat plate was studied by Anjali Devi and Julie Andrews [30]. Vajravelu et al. [31] studied the effect of convective heat transfer in the flow of viscous Ag-water and Cu-water nanofluids over a stretching surface and it was seen that the role of nanoparticle volume fraction on the flow and heat transfer characteristics under the influence of thermal buoyancy and temperature dependent internal heat generation and absorption.

Hady et al. [32] investigated the effects of thermal radiation on the viscous flow of a nanofluid and heat transfer over a non-linearly stretching sheet. Recently, Kalidas Das [33] analysed slip flow and the convective heat transfer of nanofluids over a permeable stretching surface. CVFEM has been applied to investigate flow and heat transfer of CuO water nanofluid in presence of magnetic field by Sheikholeslami et al.[34]. Unsteady two-dimensional stagnation point flow of a nanofluid over a stretching sheet is investigated numerically analysed by Malvandi et al.[35].

In this study, our objective is to investigate the effect of injection on the flow of electrically conducting viscous nanofluid past a permeable stretching surface. Here the two nanofluids considered are Copper-water nanofluid and Alumina-water nanofluid. Using an appropriate similarity transformation, the well-known governing partial differential equations are reduced to the ordinary differential equations. Numerical solution of the problem is obtained using Nachtsheim-swigert shooting iteration scheme for satisfaction of asymptotic boundary conditions along with Fourth order Runge-Kutta integration method. These numerical results for various physical parameters involved in the problem are demonstrated graphically. The skin friction coefficient and the non-dimensional rate of heat transfer are also presented numerically in tabular form for several values of the physical parameters.

2. Formulation of the Problem

Consider the steady, nonlinear hydromagnetic, two-dimensional boundary layer slip flow and heat transfer of an incompressible electrically conducting viscous water based nanofluids containing two types of nanoparticles, Copper and Alumina, past a vertical stretching surface with injection in the presence of heat sink. It is assumed that the base fluid and the nanoparticles are in thermal equilibrium and no slip occurs between them.

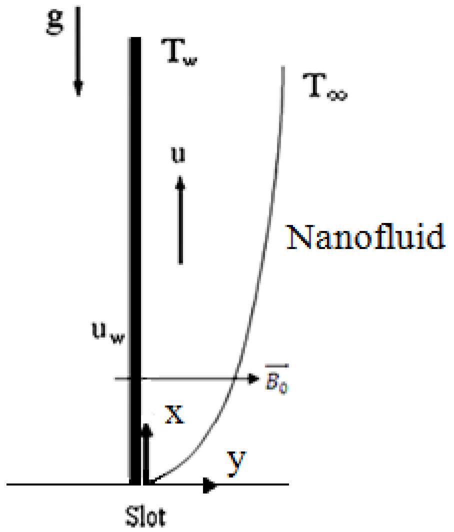

Figure 1: Physical model with Cartesian coordinate system

A uniform magnetic field of magnitude $\vec{B_0}$ is applied normal to the vertical plate. It is also assumed that the induced magnetic field is negligible and $\vec{E}$ is assumed as zero as $\text{curl}\,\vec{E}=0$ and $\text{div}\,\vec{E}=0$ is assumed as zero. Cartesian co-ordinate system is chosen. x is chosen in the vertical direction and y axis is normal to it. The plate is stretching in the vertical direction and is subjected to injection. A schematic representation of the physical model and Cartesian coordinate system is depicted in Fig. 1.

Taking the above assumptions into consideration, the steady two dimensional hydromagnetic boundary layer equations governing the convective MHD slip flow and heat transfer for a nanofluid past a vertical stretching porous surface in the presence of injection can be written as

Continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Momentum equation

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2}+g\frac{(\rho\beta)_{nf}}{\rho_{nf}}(T-T_\infty)-\frac{\sigma u B_\circ^2}{\rho_{nf}} \tag{2}$$

Energy equation

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\left(\frac{\partial^2 T}{\partial y^2}\right)+\frac{Q}{(\rho C_p)_{nf}}(T-T_\infty) \tag{3}$$

The associated boundary conditions are

$$\text{At } y=0:\ u=u_w+u_s,\ v=v_0,\ T=T_w(x)=T_\infty+\frac{Ax}{l}$$

$$\text{As } y\to\infty:\ u\to 0,\ T\to T_\infty \tag{4}$$

where

u and v are the velocity components in x and y directions respectively,

T is the temperature of the nanofluid

g is the acceleration due to gravity

Q is the volumetric rate of heat sink

μ_{nf} is the dynamic viscosity of the nanofluid

ρ_{nf} is the effective density of the nanofluid

β_{nf} is the Thermal expansion coefficient of the nanofluid

α_{nf} is the Thermal diffusivity of the nanofluid

$(\rho C_p)_{nf}$ is the heat capacitance of the nanofluid

and the expressions for $\mu_{nf},\rho_{nf},\alpha_{nf},\frac{k_{nf}}{k_f},(\rho C_p)_{nf}$ and $(\rho\beta)_{nf}$ are given through the following lines.

$$\mu_{nf}=\frac{\mu_f}{(1-\phi)^{2.5}} \tag{5}$$

$$\rho_{nf}=(1-\phi)\rho_f+\phi\rho_s \tag{6}$$

$$\alpha_{nf}=\frac{k_{nf}}{(\rho C_p)_{nf}} \tag{7}$$

$$\frac{k_{nf}}{k_f}=\frac{(k_s+2k_f)-2\phi(k_f-k_s)}{(k_s+2k_f)+\phi(k_f-k_s)} \tag{8}$$

$$(\rho C_p)_{nf}=(1-\phi)(\rho C_p)_f+\phi(\rho C_p)_{nf} \tag{9}$$

And

$$(\rho\beta)_{nf}=(1-\phi)(\rho\beta)_f+\phi(\rho\beta)_s \tag{10}$$

here

μ_f is the viscosity of the base fluid

ϕ is the solid volume fraction of nanoparticles

ρ_f is the reference density of the base fluid

ρ_s is the reference density of the nanoparticles

k_{nf} is the thermal conductivity of the nanofluid

k_f is the thermal conductivity of the base fluid

k_s is the thermal conductivity of the nanoparticles

C_p is the specific heat at constant pressure.

u_w is the velocity of the stretching surface,

u_s is the slip velocity

and v_0 is the injection velocity

where

$$u_w=ax$$

$$u_s=l\left(\frac{\partial u}{\partial y}\right)_{y=0}$$

where l is the slip length.

3. Flow Analysis

The stream function ψ satisfies the continuity equation (1) automatically with

$$u=\frac{\partial\psi}{\partial y}\qquad v=-\frac{\partial\psi}{\partial x}$$

The following dimensionless variables,

$$\eta=\left(\frac{a}{\nu_f}\right)^{\frac{1}{2}}y,\quad \psi=\left(a\nu_f\right)^{\frac{1}{2}}x\,f(\eta),$$

$$u=axf'(\eta),\quad v=-(a\nu_f)^{1/2}f(\eta)$$

$$\text{and}\quad \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty} \tag{11}$$

are introduced into the governing equations (2) and (3) together with the boundary conditions in (4) and the transformed momentum and energy equations together with the boundary conditions can be written as

$$f''' - (1-\varphi)^{2.5}\left(1-\varphi+\varphi\left(\frac{\rho_s}{\rho_f}\right)\right)(f'^2 - ff'') + (1-\varphi)^{2.5}\left(\lambda\theta\left(1-\varphi+\varphi\left(\frac{(\rho\beta)_s}{(\rho\beta)_f}\right)\right) - M^2 f'\right) = 0 \quad (12)$$

$$\frac{1}{\Pr}\frac{k_{nf}}{k_f}\theta'' - \left(1-\phi+\phi\frac{(\rho C_p)_s}{(\rho C_p)_f}\right)(f'\theta - f\theta') - \beta\theta = 0 \quad (13)$$

with boundary conditions

$$f = f_w, \quad f' = 1 + kf'', \quad \theta = 1 \quad \text{at} \quad \eta = 0$$

$$f' \to 0, \quad \theta \to 0 \quad \text{as} \quad \eta \to \infty$$

where

$\lambda = \dfrac{g(\rho\beta)_f}{\rho_f a^2}$ is the buoyancy or free convection parameter

$M^2 = \sqrt{\dfrac{\sigma B_0^2}{a\rho_f}}$ is the Magnetic interaction parameter

$\Pr = \dfrac{\nu_f}{\alpha_f}$ is the Prandtl Number

$\beta_1 = \dfrac{Q}{a(\rho C_p)_f}$ is the heat sink parameter

$Fw = \dfrac{-v_0}{(a\nu_f)^{\frac{1}{2}}}$ is the injection parameter

$k = l\left(\dfrac{a}{\nu_f}\right)^{\frac{1}{2}}$ is the slip parameter

Skin Friction Coefficient

The skin friction co-efficient C_f is given by

$$C_f = \frac{\tau_w}{\rho_f U_w^2}, \quad \text{where} \quad \tau_w = \mu_{nf}\left(\frac{\partial u}{\partial y}\right)_{y=0}$$

Using (11) we get, $C_f \operatorname{Re}_x^{1/2} = \dfrac{1}{(1-\phi)^{2.5}} f''(0)$

where Re_x is the local Reynolds Number.

Nusselt Number

The Nusselt number is defined as,

$$Nu = \frac{xq_w}{k_f(T_w - T_\infty)},$$

where $q_w = -k_{nf}\left(\dfrac{\partial T}{\partial y}\right)_{y=0}$

Using (11) we get,

$$Nu \operatorname{Re}_x^{-1/2} = \frac{-k_{nf}}{k_f}\theta'(0)$$

Table 1 represents the physical properties such as density, specific heat capacity, thermal conductivity, thermal diffusivity and thermal expansion coefficient of the base fluid and nanoparticles (Copper, Alumina).

Table 1: Physical properties of base fluid water, Copper and alumina at 25° C

	ρ (Kg/m^3)	C_p (J/Kg.K)	K (W/m.k)
Water	997.1	4179	0.613
Copper	8933	385	400
Aluminum oxide	3970	765	46

4. Numerical Solution

The numerical solution of the problem of MHD slip flow and convective heat transfer of nanofluids over a permeable vertical stretching surface has been obtained. Two cases of nanofluids such as Copper-Water nanofluid and Alumina-Water nanofluid are considered for this study. Here the problem is solved using Nachtsheim-Swigert shooting iteration technique for satisfaction of asymptotic boundary conditions along with Fourth Order Runge-Kutta Integration Method by fixing several values for the physical parameters. Numerical solutions are obtained for different values of physical parameters and their effects over velocity and temperature are analysed in detail.

5. Results and Discussion

In this section, we consider two types of water based nanofluids containing Copper (Cu) and Alumina (Al_2O_3). The Prandtl Number of the base fluid (Water) is kept constant at 7.0 and the effect of solid volume fraction φ is investigated in the range of $0 \leq \phi \leq 0.2$. In order to get the clear insight of the problem, numerical values of the solutions are obtained by fixing various values for the physical parameters involved having ϕ = 0.0 for base fluid and ϕ = 0.1 for the nanofluid when λ = 1.0 and β_1 = 0.5. The results are exhibited through the graphs and tables.

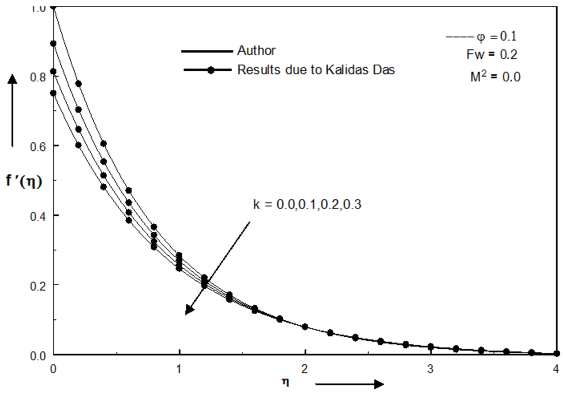

Figure 2: A comparative study of dimensionless streamwise velocity for various values of k (For Copper-water nanofluid)

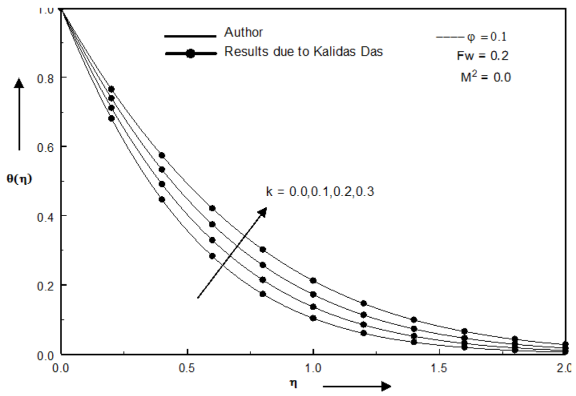

Figure 3: A comparative study of Temperature distribution for various values of k
(For Copper-water nanofluid)

In the absence of Magnetic field, the results agreed perfectly with reported by Kalidas Das (2012). This is shown through the comparison graphs in Fig. 2 and Fig. 3 which illustrate the effect of k over the flow field for Copper-Water nanofluid. Similar results are observed for Alumina-Water nanofluid also.

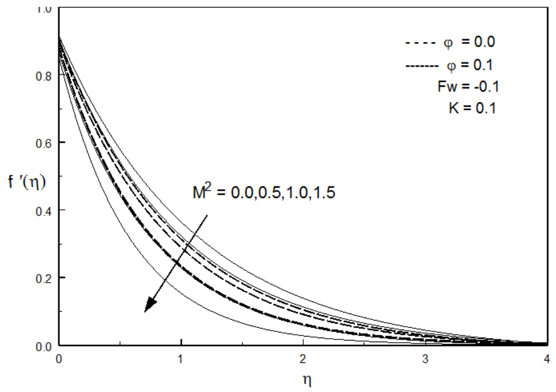

Figure 4: Velocity profiles for various values of M^2 (For Copper-water nanofluid)

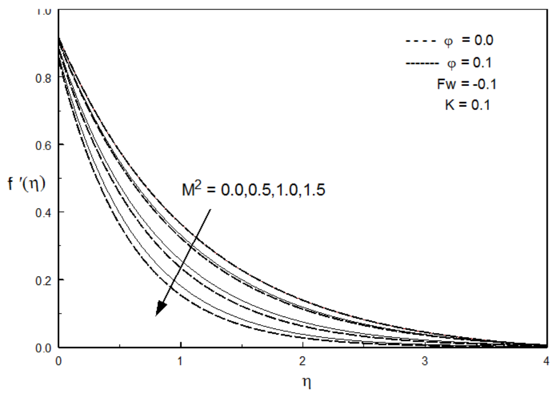

Figure 5: Dimensionless streamwise velocity profiles for various values of M^2 (For Alumina-water nanofluid)

The influence of Magnetic field on the velocity for Copper-Water nanofluid and Alumina-Water nanofluid is shown in Fig. 4 and Fig. 5 for constant values of ϕ, K, F_w. It is seen from the figures that the velocity decreases with the increase in M^2. The effect of magnetic field over temperature for Copper-Water nanofluid is shown through Fig. 6. The temperature in the boundary layer increases with the increase in the Magnetic induction parameter M^2. Same trend is followed in Alumina-Water nanofluid also and this is displayed through Fig. 7. The thermal boundary layer thickness as shown in these figures increases with an increase in thermal conductivity. It clearly indicates that the transverse magnetic field opposes the transport phenomena. This is due to the fact that the variation of M^2 leads to the variation of Lorentz force due to magnetic field and it produces more resistance to the transport phenomena.

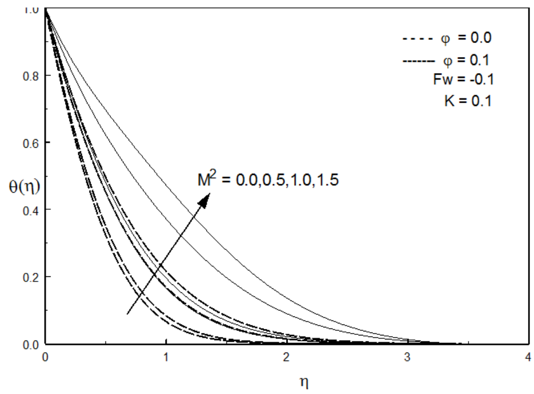

Figure 6: Temperature distribution for various values of M^2
(For Copper-water nanofluid)

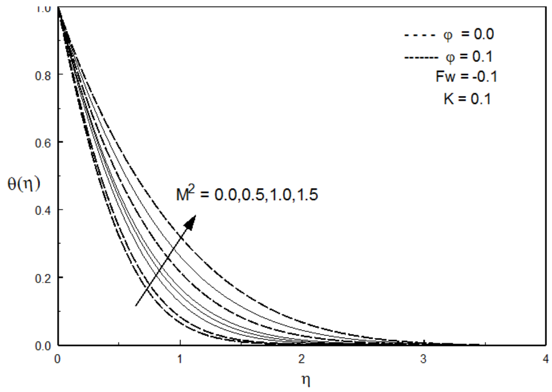

Figure 7: Temperature distribution for various values of M^2
(For Alumina-water nanofluid)

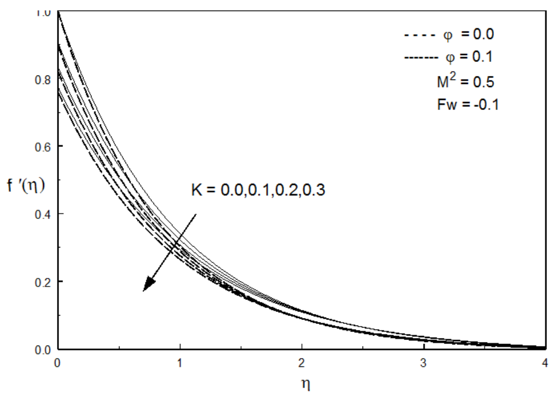

Figure 8: Velocity profiles for various values of k (For Copper-water nanofluid)

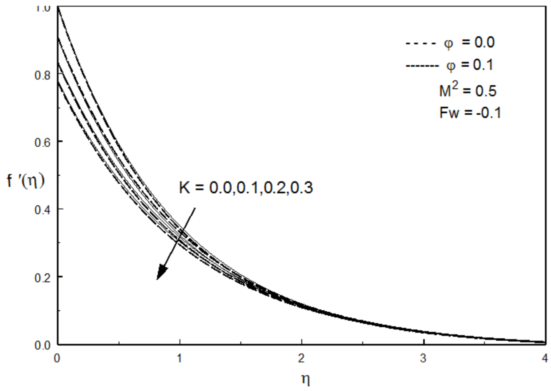

Figure 9: Dimensionless streamwise velocity for various values of k (For Alumina-water nanofluid)

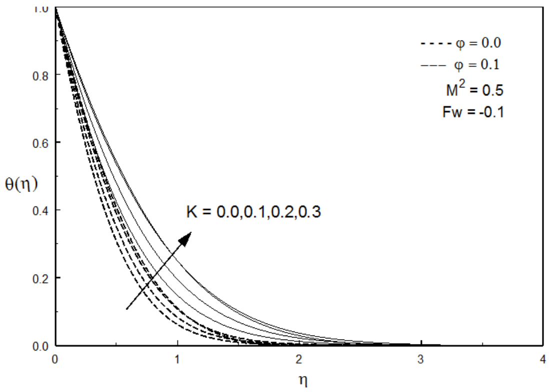

Figure 10: Temperature distribution for various values of k (For Copper-water nanofluid)

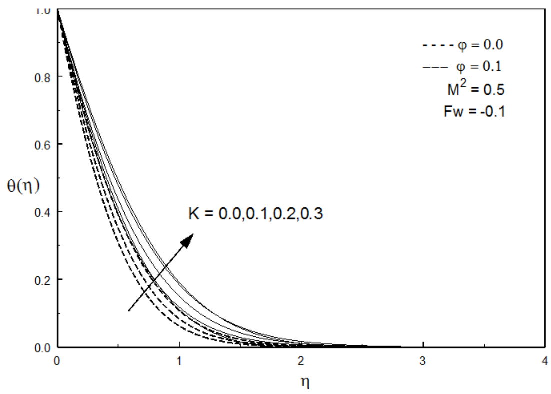

Figure 11: Temperature distribution for various values of k (For Alumina-water nanofluid)

Figs. 8-11 illustrate the velocity and temperature distributions for different values of the slip parameter k for both the Nanofluids with Nanoparticles, say Cu and Al_2O_3. Fig. 8 and 9 demonstrate that the effect of increasing value of slip parameter k is to shift the streamlines toward stretching boundary and thereby reduce thickness of the momentum boundary layer. It is also seen that the effect of slip parameter together with the presence of nanoparticles is more significant in the case of Copper – Water Nanofluid than that of Alumina – Water Nanofluid. The velocity curves show that the rate of transport decreases with the increasing distance (η) normal to the sheet. In all cases the velocity vanishes at some large distance from the sheet (at η = 4). Fig. 10 and 11 portray the effect of slip parameter on temperature distribution. The effect of slip parameter k is seen to increase the temperature. Also it is noted that at each value of k the thickness of the thermal boundary layer for Alumina – Water Nanofluid is greater than that in base fluid.

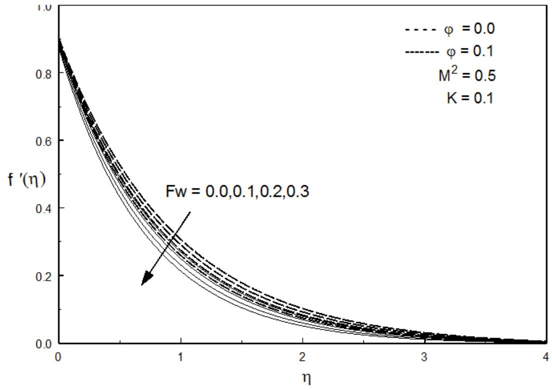

Figure 12: Velocity for various values of injection parameter (For Copper-water nanofluid)

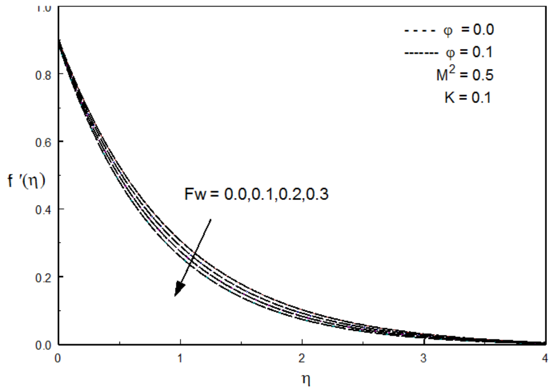

Figure 13: Velocity for various values of injection parameter (For Alumina-water nanofluid)

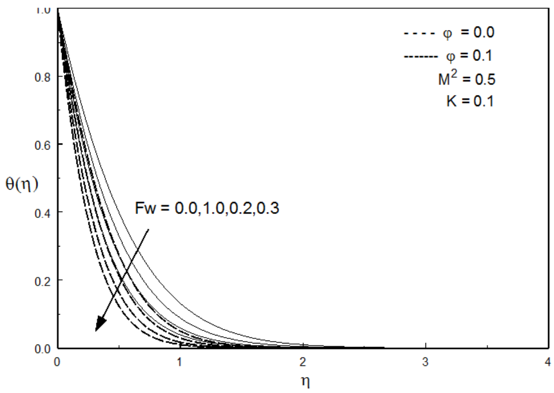

Figure 14: Temperature distribution for various values of injection parameter (For Copper-water nanofluid)

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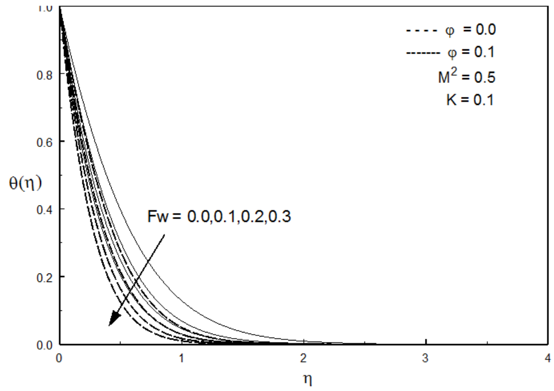

Figure 15: Velocity for various values of injection parameter (For Alumina-water nanofluid)

The effect of injection parameter (F_w) over non-dimensional streamwise velocity for Copper-Water nanofluid is shown in Fig. 12. It is clear that for increasing values of injection parameter, the velocity decelerates for both base fluid and nanofluids. Same trend is observed in the case of Alumina-Water nanofluid and this is portrayed in Fig. 13. Fig.14 displays the effect of injection parameter over the temperature for Copper-Water nanofluid and it is noted that temperature decreases as the values of injection parameter increases. Fig. 15 shows the variation in the temperature profile for Alumina-Water nanofluid for different values of injection parameter. It reveals that the temperature decreases when the injection parameter increases.

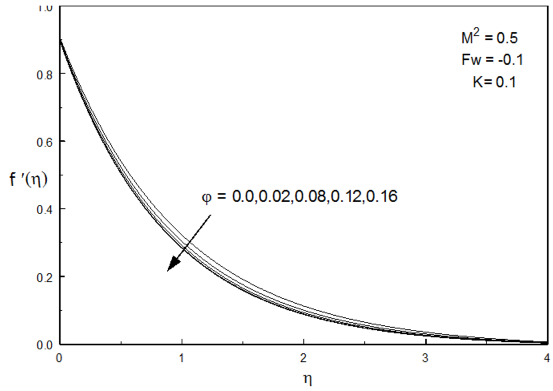

Figure 16: Effect of volume fraction over Velocity (For Copper-water nanofluid)

Table 3: Skin Friction Coefficient and Non-dimensional rate of heat transfer for Copper Water Nanofluid and Alumina Water Nanofluid for different ϕ, M^2, k and F_w

ϕ	M^2	k	F_w	Copper-Water nanofluid $\frac{1}{(1-\phi)^{2.5}} f''(0)$	Copper-Water nanofluid $-\frac{k_{nf}}{k_f}\theta'(0)$	Alumina-Water nanofluid $\frac{1}{(1-\phi)^{2.5}} f''(0)$	Alumina-Water nanofluid $-\frac{k_{nf}}{k_f}\theta'(0)$
0.0	0.0	0.1	0.1	-0.83643	1.80358	-0.83643	1.80358
	0.5			-0.93217	1.71673	-0.93217	1.71673
	1.0			-1.16856	1.53288	-1.16856	1.53288
	1.5			-1.26209	1.46509	-1.26209	1.46509
0.0	0.5	0.0	0.1	-1.07029	1.91479	-1.07029	1.91479
		0.1		-0.93217	1.71673	-0.93217	1.71673
		0.2		-0.82909	1.54488	-0.82909	1.54488
		0.3		-0.72573	1.44869	-0.72573	1.44869
0.0	0.5	0.1	0.0	-0.97069	2.12179	-0.97069	2.12179
			0.1	-1.01069	2.57110	-1.01069	2.57110
			0.2	-1.05213	3.05816	-1.05213	3.05816
			0.3	-1.09496	3.57692	-1.09496	3.57692
0.1	0.0	0.1	0.1	-1.23976	1.85816	-1.08740	2.00269
	0.5			-1.32197	1.75745	-1.28445	1.90664
	1.0			-1.53947	1.72969	-1.43363	1.82688
	1.5			-1.57286	1.67458	-1.62680	1.74547
0.1	0.5	0.0	0.1	-1.54555	2.02484	-1.35794	2.13432
		0.1		-1.32197	1.75745	-1.18445	1.90664
		0.2		-1.16090	1.51542	-1.05463	1.70550
		0.3		-0.99095	1.44235	-0.91866	1.60921
0.1	0.5	0.1	0.0	-1.38884	2.16919	-1.23461	2.31173
			0.1	-1.45885	2.61588	-1.28675	2.75385
			0.2	-1.53191	3.09394	-1.34081	3.22835
			0.3	-1.60787	3.59943	-1.39675	3.73075

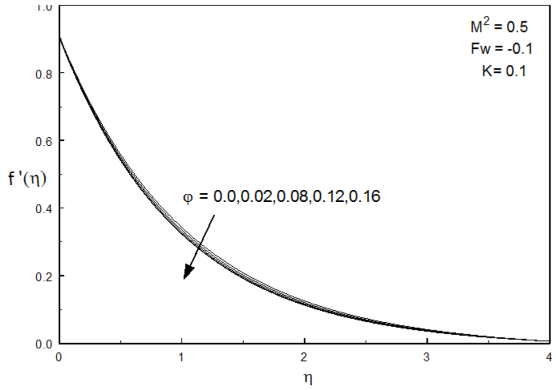

Figure 17: Effect of volume fraction over non-dimensional streamwise velocity (For Alumina-water nanofluid)

Table 2: Effect of solid volume fraction over $\frac{1}{(1-\phi)^{2.5}} f''(0)$ and $-\frac{k_{nf}}{k_f}\theta'(0)$ for two nanofluids when $k = 0.1$, $F_w = 0.1$ and $M^2 = 0.5$

ϕ	Copper-Water nanofluid		Alumina-Water nanofluid	
	$\frac{1}{(1-\phi)^{2.5}} f''(0)$	$-\frac{k_{nf}}{k_f}\theta'(0)$	$\frac{1}{(1-\phi)^{2.5}} f''(0)$	$-\frac{k_{nf}}{k_f}\theta'(0)$
0.00	-0.93217	1.71673	-0.93217	1.71673
0.02	-1.00673	1.72859	-0.97807	1.75191
0.04	-1.08242	1.73899	-1.02605	1.78669
0.06	-1.15985	1.74751	-1.07631	1.82114
0.08	-1.23953	1.75375	-1.12903	1.85537
0.10	-1.32197	1.75745	-1.18445	1.88950

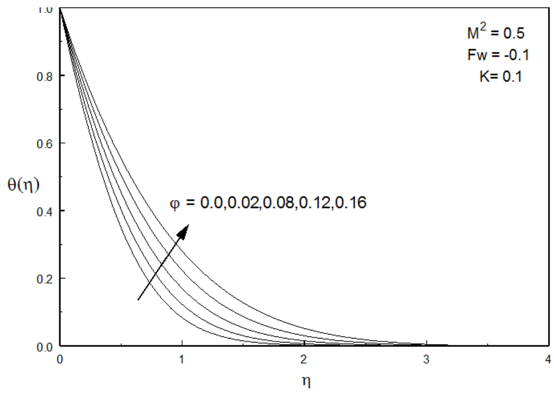

Figure 18: Temperature distribution for various values of ϕ (For Copper-water nanofluid)

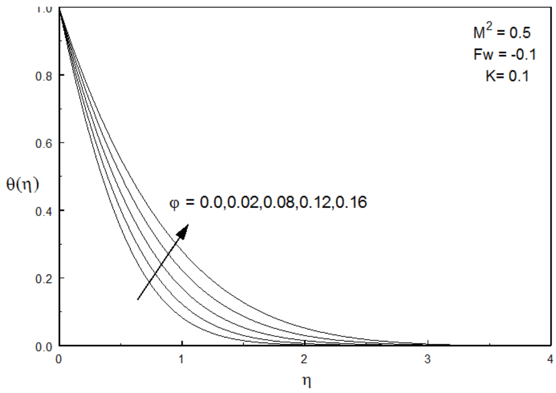

Figure 19: Temperature distribution for various values of ϕ (For Alumina-water nanofluid)

The effect of solid volume fraction (ϕ) of nanoparticles over the velocity for Copper-Water nanofluid is displayed in Fig. 16 and the velocity decreases as the solid volume fraction increases. Same trend is observed in the case of Alumina-Water nanofluid. But the effect is less significant in this case as shown in Fig. 17. Fig. 18 illustrates the temperature distribution for Copper-Water nanofluid for different values of solid volume fraction ϕ. It is seen that the increasing values of ϕ increases the temperature. The effect of solid volume fraction over the temperature for Alumina-Water nanofluid is displayed in Fig. 19. Even in the case of Alumina-Water nanofluid, the temperature is increases for higher values of ϕ.

The influence of the solid volume fraction on the skin friction coefficient and non-dimensional rate of heat transfer are given in Table 2. It is observed from this table that the skin friction coefficient and non-dimensional rate of heat transfer increases for both nanofluids with an increase in ϕ.

Table 3 depicts the skin friction coefficient and dimensionless rate of heat transfer for both nanofluids for different values of M^2, k, F_w and ϕ. For both the nanofluids the skin friction coefficient increases in magnitude when the magnetic interaction parameter and injection parameter increases. Whereas, increase in M^2 and F_w decreases the dimensionless rate of heat transfer for both the nanofluids. As slip parameter increases, skin friction coefficient reduces and non-dimensional rate of heat transfer is decreased.

6. Conclusion

This work presents an analysis of the combined effects of viscous dissipation, injection and solid volume fraction on the hydromagnetic two-dimensional boundary layer of Water – based two nanofluids over a permeable vertical stretching sheet in the presence of heat absorption. The equations of motion and energy which are non-linear partial differential equations are reduced to non-linear ordinary differential equations by means of similarity transformations. The resulting non-linear ordinary differential equations are solved numerically for various values of physical parameters. Two types of nanofluids were considered, Copper-Water and Alumina-Water, and our results revealed the following:

- In general, the results of this study are in complete agreement with that of Kalidas Das [27] in the absence of Magnetic field.
- Both the fluid velocity and the hydrodynamic boundary layer thickness decrease with increase in M^2, ϕ, k and F_w.
- The temperature enhances with the increasing values of M^2, ϕ, k and decreases with F_w for both the Nanofluids.
- Increase in Magnetic interaction parameter, Injection parameter and volume fraction increases the Skin friction coefficient in magnitude whereas increasing slip parameter decreases the Skin friction coefficient in magnitude.
- Increase in volume fraction and Injection parameter increases the non-dimensional rate of heat transfer while the same is decreased due to increasing values of magnetic interaction parameter and slip parameter.
- The rate of heat transfer at the surface is higher for nanofluid than that for regular fluid.

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