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# Comparison of Bending Response of Laminated Composite Plates under Mechanical and Hygro-Thermal Loading

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**Abstract:** ***In recent years many researches in deformation and stress analysis of laminated composite plate subjected to moisture and temperature has been conducted but the study has focused mainly in the effect of the temperature on the composite plate. Temperature and moisture variation is always an important factor to be considered in designing any parts of an aircraft, which may sometimes lead to catastrophic effects if these are not given proper consideration while designing. Composites are subjected to varied environmental conditions during the service life. Moisture and temperature have an unfavorable effect on the performance of composites. Stiffness and strength are reduced in proportion to increase in moisture concentration and temperature. In the present investigation, the quadratic iso-parametric plate bending element is applied to study the effect of moisture and temperature on the bending behavior of laminated composite plates. A Matlab code is generated to first find the deflection and stress in a composite plate when the environmental effect is not considered and in second case the variation of moisture and temperature is taken into account and its deflection and moments are found out and compared with the open source research journals. Reduced lamina material properties at elevated moisture concentration and temperature are used in the analysis. Deflections and stress resultants are evaluated for angle-ply and anti-symmetric cross ply laminates at different moisture concentrations and temperatures for clamped and simply-supported boundary conditions and their distribution in the plate is studied. In case of anti-symmetric angle-ply laminates, fiber orientation is also considered.***

**Keywords:** Composite structures, Finite Element method, Hygrothermal Loading, Laminated composite plates, Bending Analysis

## 1. Introduction

Since the discovery of composite material, there has been continuously increasing research and development efforts in the area of manufacturing, testing, modeling, characterization of composite materials.

David Roylance[1] has outline the mechanics of fiber reinforced laminated plates, leading to a computational scheme that relates the in-plane strain and curvature of a laminate to the tractions and bending moments imposed on it. B.N. Pandya & T. Kant[2] presents his idea of a $C^0$ continuous displacement finite element formulation of a higher order theory for flexure of thick arbitrary laminated composite plates under transverse load is presented. Avinash Ramsaroop, Krishnan Kanny's[3] work deal with the generation of MATLAB script files that assists the user in the design of a composite laminate to operate within safe conditions. The inputs of the program are the material properties, material limits and loading conditions. Wu and Tauchert[4] presented closed-form solutions for deflections and stress resultants for symmetric and anti-symmetric laminates subjected to temperature in addition to the external loading. Debabrata Gayen, Tarapada Roy's[5] works deals with an analytical method in order to determine the stress distributions (such as axial in-plane stresses and inter-laminar shear stresses) in multilayered symmetric and anti-symmetric circular tapered laminated composite beams under hygro and thermal loadings. Ashraf M. Zenkour[6] presented the sinusoidal shear deformation plate theory which is used to study the response of multilayered angle-ply composite plates due to a variation in temperature and moisture concentrations.

Pipes et al.[7] presented the distribution of in-plane stresses across the thickness for the symmetric laminates subjected to moisture absorption and desorption. Hui-shenshen[8] investigated the influence of hygrothermal effects on the nonlinear bending of shear deformable laminated plates subjected to a uniform or sinusoidal load using a micro-to-micromechanical analytical model. SY Lee, JL Jang, Jeng sheng lin and CJ Chou[9] studied the influence of hygrothermal effects on the cylindrical bending of a pinned-pinned supported symmetric angle ply laminated plate subjected to a uniform transverse load is evaluated via the CLPT and von Karman larger deflection theory, respectively.

S.K. Singh, A. Chakrabarti[10] has presented a formulation in which the plate model has been implemented with a computationally efficient $C^0$ finite element developed by using consistent strain field. Special steps are introduced to circumvent the requirement of $C^1$ continuity in the original plate formulation and $C^0$ continuity of the present element has been compensated in stiffness matrix calculations. The accuracy of the proposed $C^0$ element is established by comparing the results with those obtained by three dimensional elasticity solutions and other finite element analysis. B.P. Patel, M. Ganapathi, D.P. Makhecha[11] have formulated theory which accounts for the nonlinear variation of the in-plane and transverse displacements through the thickness, and abrupt discontinuity in slope of the in-plane displacements at any interface. The analysis is carried out employing a $C^0$ QUAD-8 iso-parametric higher-order finite

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element. Navier, Levy, and Rayleigh-Ritz developed solutions to composite beams and plate problems. However, exact analytical or variational solutions to these problems cannot be developed when complex geometries, arbitrary boundary conditions or nonlinearities are involved. Therefore one must resort to approximate methods of analysis that are capable of solving such problems. There are several theories available to describe the kinematics of the laminates. Classical Laminate Plate Theory and First order Shear Deformation Theory are some among them.

From above, it is clear that the analysis and design of mechanical behavior in composite material is of great importance in real industries. There is a critical need since composites are generally exposed to several thermal environments and, therefore it is very important to analyze the behavior of composite at different temperatures. In the context of ESL theories, the simplest one is the CLT which neglects the shear contribution in the laminate thickness. However the use of a shear deformation laminate theory is recommended for flat structures made of fiber-reinforced composite materials characterized by non-negligible shear deformations in the thickness direction, since the longitudinal elastic modulus of the lamina can much higher than the shear and the transversal moduli. The extension of the Reissner and Mindlin model to the case of laminated Anisotropic plates, i.e. FSDT, accounts for shear deformation along the thickness. It gives satisfactory results for a wide class of structural problems, even for moderately thick laminates, requiring only $C^0$-continuity for the displacement field. Shear correction factors must be introduced where the transverse shearing strains (stresses) are assumed to be constant along the plate thickness so that stress boundary conditions on the top and the bottom of the plate are violated. The determination of shear correction factors depends both on the lamination sequence and on the state of deformation.

The objective of the present work is to determine transverse displacements; normal stresses and moments of 4 layered cross ply/angle ply laminated composite square plate subjected to changes in thermal load and changes in moisture content of the environment when it is simply supported/clamped at the edges. In this present work, a displacement based finite element model is formulated based on First order Shear Deformation Theory. It is an iso-parametric element with 9 nodes and 5 degrees of freedom at each node. The 5 degrees of freedom are;

u, v - displacement along x and y direction,
w - deflection normal to the plate
Ɵx ,Ɵy - rotation about x and rotation about y axis

At first the bending analysis of laminated composite plate has been done from simple mechanical loading. The moisture coefficient and temperature coefficient strains have been added in the finite element analysis to find out the effect of moisture on deflection of laminated composite plate. Variation of deflection and various bending moment along the x axis of the plate with the change in moisture concentration and temperature coefficient has been obtained for both anti symmetric and angle ply plate for simply supported and angle ply plate and the variation of twisting moment along the orientation angle with change in moisture concentration for angle ply plate has been obtained.

## 2. Mathematical Formulation

Consider a plate of constant thickness t composed of arbitrary number of thin laminae, each oriented at an angle with the x-axis of the coordinate system. The coordinate system has the origin at the center of the plate with the z-axis perpendicular to the plane of the plate. The resultants forces and moments acting on a laminate are given by

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \sum_{K=1}^{N} \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{12} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{12} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix} \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \sum_{K=1}^{N} \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{12} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{12} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{bmatrix} \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix}$$

Where N is the force per unit length(width) and M is the moment per unit length

The constitutive equations for the plate subjected to moisture and temperature are given by

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \\ Q_x \\ Q_y \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} & 0 & 0 \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} & 0 & 0 \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} & 0 & 0 \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} & 0 & 0 \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} & 0 & 0 \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & A_{44} & A_{45} \\ 0 & 0 & 0 & 0 & 0 & 0 & A_{45} & A_{55} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ K_x \\ K_y \\ K_{xy} \\ \phi_x \\ \phi_y \end{bmatrix} - \begin{bmatrix} N_x^N \\ N_y^N \\ N_{xy}^N \\ M_x^N \\ M_y^N \\ M_{xy}^N \\ 0 \\ 0 \end{bmatrix}$$

where $N_x, N_y$ and $N_{xy}$ are in-plane force resultants; $M_x, M_y$ and $M_{xy}$ are the moment resultants $Q_x$ and $Q_y$ are transverse shear resultants. $N_x^N, N_y^N$ and $N_{xy}^N$ are non-mechanical force resultants and $M_x^N, M_y^N$ and $M_{xy}^N$ are the non-mechanical moment resultants due to the moisture and temperature. They are given by

$$\{N_x^N, N_y^N, N_{xy}^N\}^T = \sum_{k=1}^{n} \int_{z_k-1}^{z_k} (\overline{Q}_{ij})_k \{e_k\} dz$$

$$\{M_x^N, M_y^N, M_{xy}^N\}^T = \sum_{k=1}^{n} \int_{z_k-1}^{z_k} (\overline{Q}_{ij})_k \{e_k\} zdz$$

where i, j=1,2,6, $z_{k-1}$ and $z_k$ are the layer distances and $\{e\}_k = \{e_x, e_y, e_{xy}\}_k^T$ are non-mechanical strains of a laminae which are expressed as

$$\begin{bmatrix} e_x \\ e_y \\ e_{xy} \end{bmatrix}_k = \begin{bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{bmatrix}_k (C - C_0) + \begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{bmatrix}_k (T - T_0)$$

where $\beta_x$, $\beta_y$, $\beta_{xy}$ are moisture coefficients of the lamina obtained by transformation from $\beta_1$ and $\beta_2$. $\alpha_x$, $\alpha_y$ , $\alpha_{xy}$ are thermal coefficients obtained in the same manner from $\alpha_1$, $\alpha_2$. $C_0$ and $T_o$ are the reference moisture concentration and temperature, and C and T are the elevated moisture concentration and temperature, respectively, which are in general, functions of x, y, and z. The stiffness coefficients are defined as

$$(A_{ij}, B_{ij}, D_{ij}) = \sum_{k=1}^{n} \int_{z_k-1}^{z_k} (\overline{Q_{ij}})_k (1, z, z^2) dz \qquad i,j=1,2,6$$

$$(A_{ij}) = \alpha \sum_{k=1}^{n} \int_{z_k-1}^{z_k} (\overline{Q_{ij}})_k dz \qquad i,j=4,5$$

where α is the shear correction factor

$$(Q_{ij})_k = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \qquad i,j=1,2,6$$

$$(Q_{ij})_k = \begin{bmatrix} Q_{44} & 0 \\ 0 & Q_{55} \end{bmatrix} \qquad i,j=4,5$$

Where

$$Q_{11} = E_1 / (1-\nu_{12}\nu_{21}),$$
$$Q_{12} = \nu_{12} E_2 / (1-\nu_{12}\nu_{21}),$$
$$Q_{22} = E_2 / (1-\nu_{12}\nu_{21}),$$
$$Q_{44} = G_{13}, Q_{55} = G_{23}$$
$$\varepsilon_x^0 = \frac{\partial u^0}{\partial x}, \varepsilon_y^0 = \frac{\partial v^0}{\partial y}, \gamma_{xy}^0 = \frac{\partial u^0}{\partial y} + \frac{\partial v^0}{\partial x},$$
$$K_x = \frac{\partial \theta_y}{\partial x}, K_y = -\frac{\partial \theta_x}{\partial y}, K_{xy} = \frac{\partial \theta_y}{\partial y} - \frac{\partial \theta_x}{\partial x},$$
$$\phi_x = \theta_y + \frac{\partial w}{\partial x}, \phi_y = -\theta_x + \frac{\partial w}{\partial y},$$

Final equation is

$$\{F\} = [D]\{\varepsilon_p\} - \{F^N\}$$

## 3. Finite Element Formulation

A nine-noded isoparametric element with five degrees of freedom at each node viz., $u^0$, $v^0$, w, $\Theta_x$ and $\Theta_y$ is used for the present analysis. The displacements are expressed in terms of their nodal values using the element shape functions.

$$u^0 = \sum_{i=1}^{9} N_i u_i^0, v^0 = \sum_{i=1}^{9} N_i v_i^0, w = \sum_{i=1}^{9} N_i w_i, \theta_x = \sum_{i=1}^{9} N_i \theta_{xi}, \theta_y = \sum_{i=1}^{9} N_i \theta_{yi}$$

The strains are obtained by

$$\{\varepsilon_p\} = [B]\{\delta_e\}$$

where [B] is matrix relating strains and displacements and $\{\delta_e\}$ is the matrix of nodal displacements of an element.

The element stiffness matrix is given by

$$[K_e] = \iint [B]^T [D][B] dxdy$$

The element level nodal load vector due to the external transverse load is obtained as

$$\{P_e\} = \iint N_i \begin{bmatrix} q \\ 0 \\ 0 \end{bmatrix} dxdy$$

where q is the transverse load per unit area. The element level nodal load vector due to the non-mechanical forces and moments is given by

$$\{P_e^N\} = \iint [B]^T \{F^N\} dxdy$$

The solution to the displacements is obtained from the equilibrium condition

$$[K]\{\delta\} = \{P\} + \{P_N\}$$

## 4. Results and Discussions

### A. When Mechanical Load is applied

For simply supported cross ply (0/90/90/0) square laminate subjected to uniformly distributed Loading of material made of Graphite/epoxy composite with properties as

$E_1$=175 GPa, $E_2$=7 GPa, $G_{12}=G_{13}=(0.5E_2)$=3.5 GPa, $G_{23}=(0.2E_2)$=1.4 GPa, $\nu_{12}$=0.25, Shear correction factor K=5/6

Element type-9 noded Isoparametric Quadratic Element

**Table 1:** Non dimensionalized maximum transverse deflection and stresses of simply supported cross-ply (0/90/90/0) square plate subjected to uniformly distributed loading

| a/t | t | Type of solution | w | $\sigma_{xx}$ | $\sigma_{yy}$ | $\sigma_{xz}$ | $\tau_{xy}$ | $\tau_{yz}$ |
|---|---|---|---|---|---|---|---|---|
| 10 | 0.1 | FEM(20x20) | 101.768 | 0.7467 | 0.489 | 0.044 | 0.789 | 0.345 |
| | | FEM(5x5) | 105.454 | 0.6847 | 0.412 | 0.039 | 0.712 | 0.276 |
| | | Analytical solution# | 102.50 | 0.7577 | 0.500 | 0.047 | 0.798 | 0.349 |
| 20 | 0.05 | FEM(20x20) | 76.89 | 0.7967 | 0.394 | 0.041 | 0.826 | 0.319 |
| | | FEM(5x5) | 78.83 | 0.7121 | 0.298 | 0.032 | 0.699 | 0.219 |
| | | Analytical solution# | 76.94 | 0.8045 | 0.396 | 0.042 | 0.830 | 0.322 |
| 100 | 0.01 | FEM(20x20) | 66.23 | 0.7869 | 0.319 | 0.031 | 0.785 | 0.293 |
| | | FEM(5x5) | 72.43 | 0.6854 | 0.219 | 0.029 | 0.684 | 0.249 |
| | | Analytical solution# | 68.33 | 0.842 | 0.355 | 0.039 | 0.842 | 0.314 |

***# Analytical solution is obtained from the book on finite element analysis by J.N. Reddy***

where

w -Non dimensionalized central transverse deflection

$\sigma_{xx}$ -Non dimensionalized normal stress $\sigma$ in xx plane

$\sigma_{yy}$ -Non dimensionalized normal stress $\sigma$ in yy plane

$\sigma_{xz}$ -Non dimensionalized shear stress in xz plane

$\tau_{yz}$ -Non dimensionalized shear stress in yz plane

$\tau_{xy}$ -Non dimensionalized normal stress in xy plane

The following non-dimensional quantities are used to get the non dimensionalized stresses and deflections from the actual ones.

*courtesy-Mechanics of laminated composite plates and shells by J.N Reddy

$$\overline{w} = w_0(0,0)\frac{E_2h^2}{a^4q_0}$$

$$\overline{\sigma_{xy}} = \sigma_{xy}(a/2, a/2, -h/2)\frac{h^2}{a^2q_0}$$

$$\overline{\sigma_{xx}} = \sigma_{xx}(0,0,h/2)\frac{h^2}{a^2q_0}$$

$$\overline{\sigma_{yy}} = \sigma_{yy}(0,0,h/4)\frac{h^2}{a^2q_0}$$

$$\overline{\sigma_{xz}} = \sigma_{xz}(a/2, 0, k=1,4)\frac{h}{aq_0}$$

$$\overline{\sigma_{yz}} = \sigma_{yz}(0, a/2, k=1,4)\frac{h}{aq_0}$$

The origin of the coordinate system is taken at the center of the plate, -a/2<(x,y)<a/2 and –h/2<z<h/2. As mentioned earlier, the stresses are computed at the reduced Gauss points.

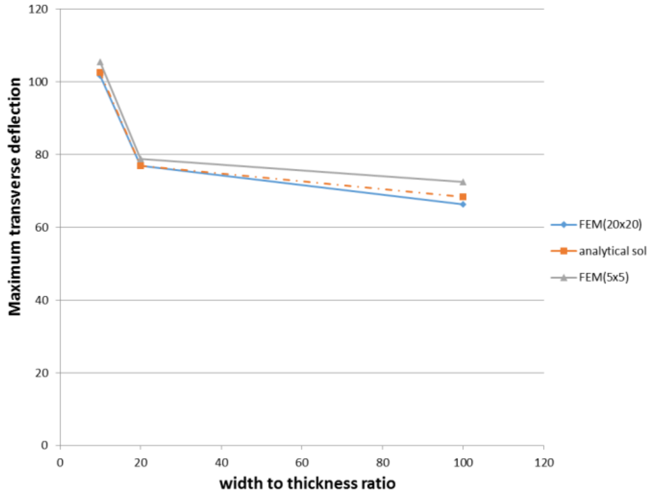


**Figure I:** Non dimensionalized transverse central deflection versus width to thickness ratio for simply supported cross ply (0/90/90/0) square laminate subjected to uniformly distributed transverse loading

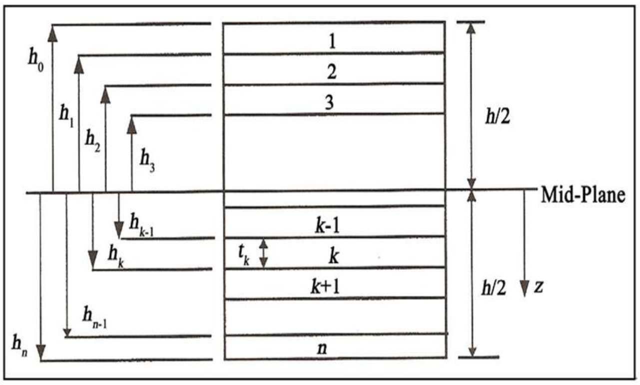


**Figure II**: Location of Layers in Composite Structure*

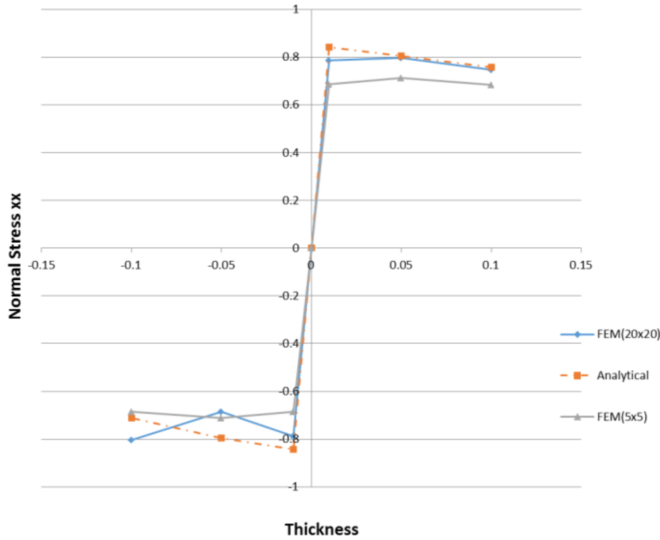


**Figure III**: Non dimensionalized normal stress $\sigma_{XX}$ versus width to thickness ratio for simply supported cross ply (0/90/90/0) square laminate subjected to uniformly distributed transverse loading

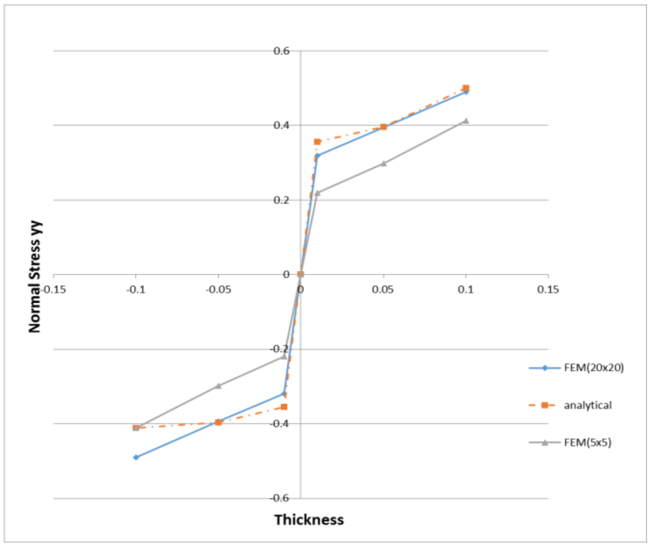


**Figure IV**

Non dimensionalized normal stress $\sigma_{YY}$ versus width to thickness ratio for simply supported cross ply (0/90/90/0) square laminate subjected to uniformly distributed transverse loading

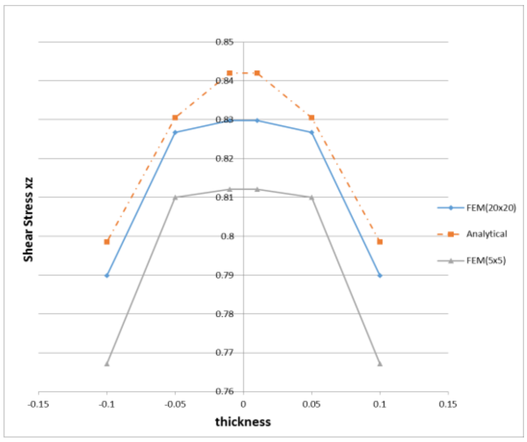


**Figure V**

Non dimensionalized transverse shear stress $\tau_{XZ}$ versus width to thickness ratio for simply supported cross ply (0/90/90/0) square laminate subjected to uniformly distributed transverse loading

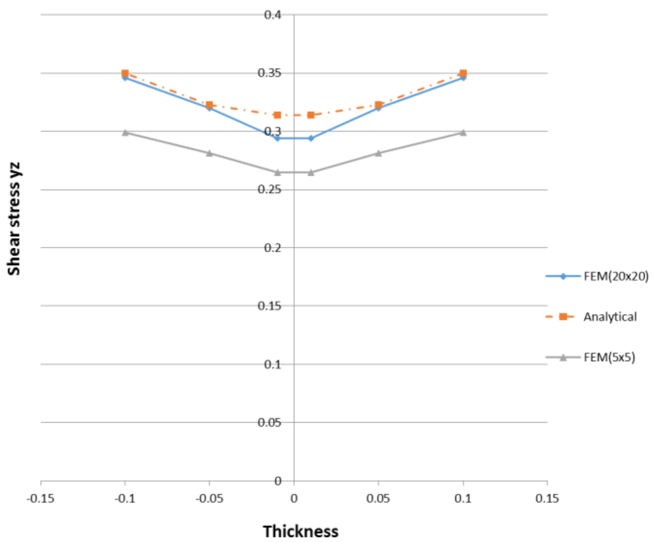


**Figure VI:** Non dimensionalized transverse shear stress $\tau_{YZ}$ versus width to thickness ratio for simply supported cross ply (0/90/90/0) square laminate subjected to uniformly distributed transverse Loading

### B. When Hygrothermal Load is applied

The results here are obtained for only change in moisture content for graphite epoxy laminate. Square laminates of side to thickness ratio of 100 are analyzed for simply supported and clamped boundary condition. The formulation and accuracy of the present finite element analysis is verified with the standard research paper "***Hygrothermal effects on the bending characteristics of laminated composite Plates" by K.S Sairam and P.K Sinha***[24]. Material Property table is obtained from the journal paper by P.K Sinha and K.S Sairam.

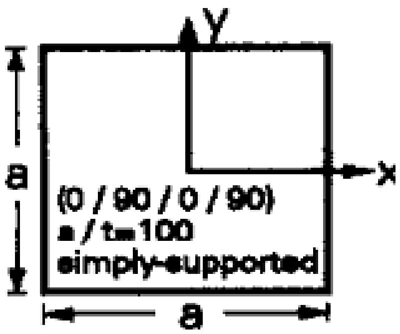


**Figure VII**

Simply Supported Laminated Plate*

*courtesy-Mechanics of laminated composite plates and shells by j.n reddy

**Table II:** Elastic Moduli of Graphite/Epoxy Lamina At Different Moisture Concentrations.

$G_{13} = G_{12}$, $G_{23} = 0.5G_{12}$, $V_{12} = 0.3$, $B_1=0$, AND $B_2= 0.44$

| | Moisture Concentration C (%) | | | | | | |
|---|---|---|---|---|---|---|---|
| **Elastic Modulus (Gpa)** | **0** | **0.25** | **0.5** | **0.75** | **1** | **1.25** | **1.5** |
| $E_1$ | 130 | 130 | 130 | 130 | 130 | 130 | 130 |
| $E_2$ | 9.5 | 9.25 | 9 | 8.75 | 8.5 | 8.5 | 8.5 |
| $G_{12}$ | 6 | 6 | 6 | 6 | 6 | 6 | 6 |

**Table III:** Stress Resultant At different Moisture Concentration (0/90/0/90) For Clamped Boundary Condition

| | Moisture Concentrations C (%) | | | | | | |
|---|---|---|---|---|---|---|---|
| Moment (Nmm) | 0 | 0.25 | 0.5 | 0.75 | 1 | 1.25 | 1.5 |
| $M_x$ | 0 | -0.428 | -0.863 | -1.256 | -1.675 | -2.068 | -2.446 |
| $M_y$ | 0 | 0.428 | 0.863 | 1.256 | 1.675 | 2.068 | 2.446 |

**Table IV:** Verification of FEM Results With Closed Form Solution Given in Journal Paper by P.K Sinha[24] At Temperature T=400 K (0/90/0/90) Simply Supported

| Bending characteristics | Solution | a/b=0 | a/b=0.33 | a/b=0.66 | a/b=1 |
|---|---|---|---|---|---|
| w(mm) | Closed form# | 0 | 0.0085 | 0.0267 | 0.0337 |
| | Present Fem | 0 | 0.008 | 0.264 | 0.0333 |
| $M_x$(Nmm) | Closed form# | -2.753 | -2.518 | -1.869 | -0.966 |
| | Present Fem | -2.793 | -2.55 | -1.875 | -0.968 |
| $M_y$ (Nmm) | Closed form# | 2.753 | 2.752 | 2.657 | 2.237 |
| | Present Fem | 2.792 | 2.776 | 2.683 | 2.282 |

# Closed Form is Obtained From The Journal Paper by P.K. Sinha and K.S. Sairam[24]

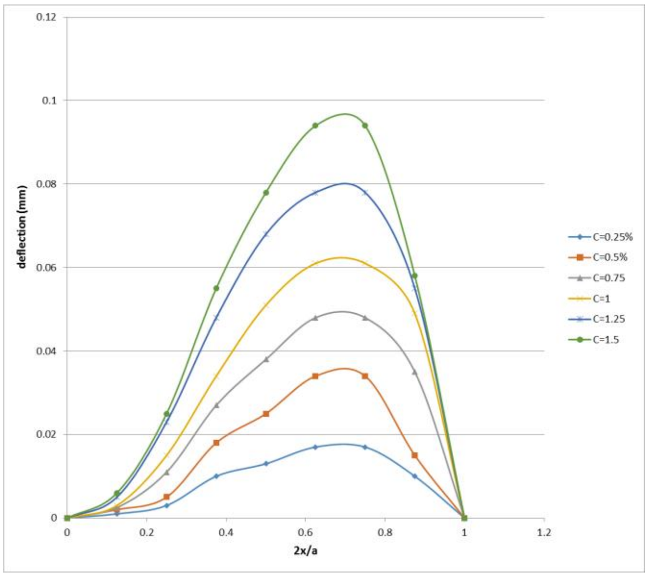


**Figure VIII**

Transverse Deflection Along X Axis For Different Moisture Concentration (Simply Supported)

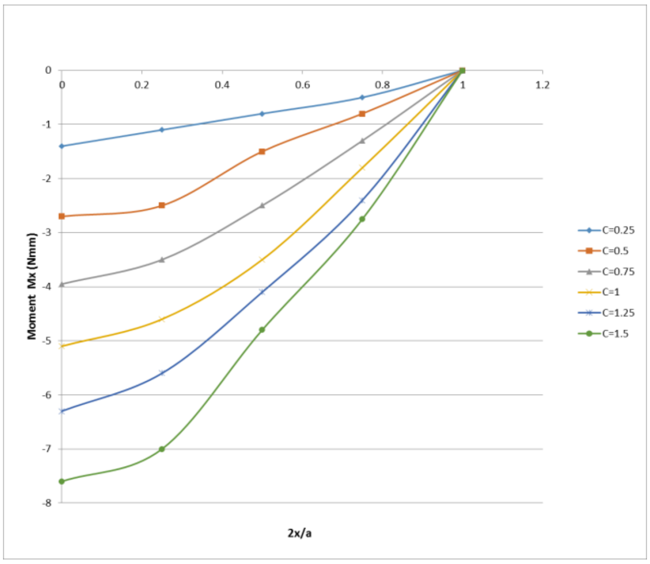


**FIGURE IX**

Moment $M_X$ For Different Moisture Concentration at Simply Supported

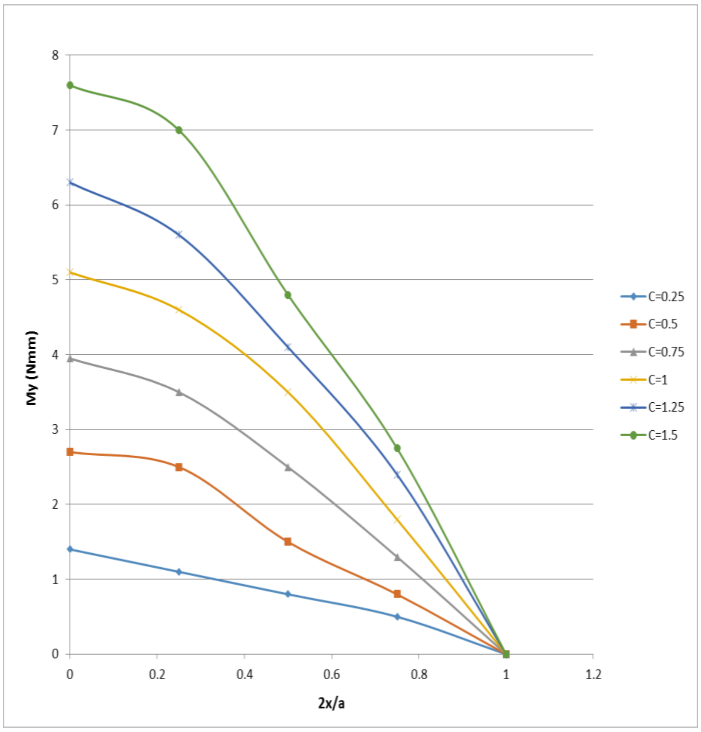


**FIGURE X**

Moment $M_Y$ For Different Moisture Concentration at Simply Supported

*courtesy-Mechanics of laminated composite plates and shells by j.n reddy

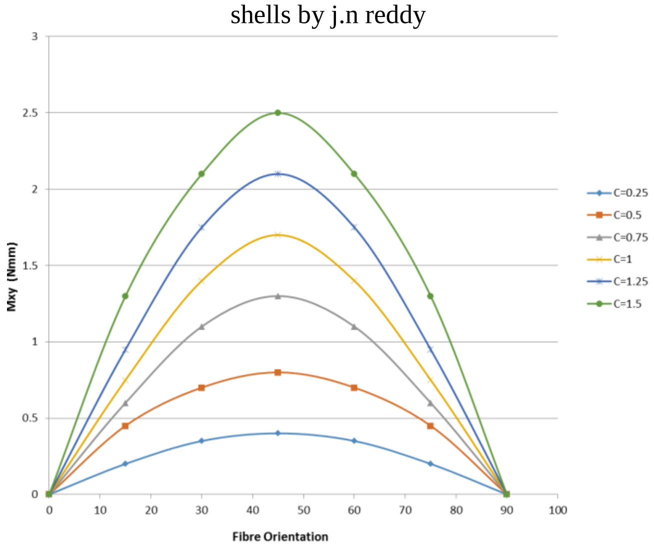


**Figure XI:** Variation of $M_{XY}$ with Fiber Orientation for Different Moisture Concentration

**Table V:** Elastic Moduli of Graphite/Epoxy Lamina At Different Temperature.

G13 = G12, G23 = 0.5G12, v12 = 0.3, β1=0, and β2= 0.44

| | Temperature T(K) | | | | | |
|---|---|---|---|---|---|---|
| Elastic | 300 | 325 | 350 | 375 | 400 | 425 |
| E1 | 130 | 130 | 130 | 130 | 130 | 130 |
| E2 | 9.5 | 8.5 | 8.0 | 7.5 | 7.0 | 6.75 |
| G12 | 6.0 | 6.0 | 5.5 | 5.0 | 4.7 | 4.50 |

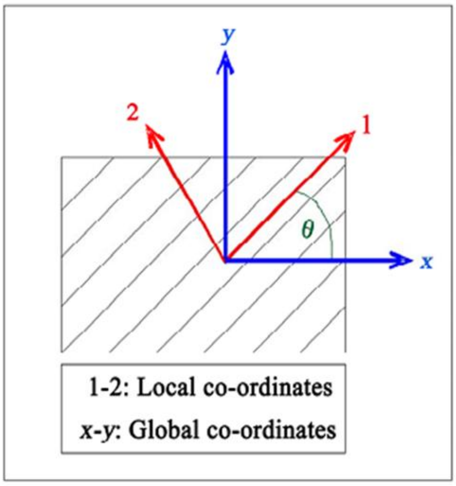


**Figure XII:** Global Co-Ordinate System In Relation To Local Coordinate System*

**Table VI**: Stress Resultant At Different Temperatures (0/90/0/90) for Clamped Boundary Condition

| Temperature T(K) | | | | | | |
|---|---|---|---|---|---|---|
| Moment( | 30 | 32 | 35 | 375 | 400 | 425 |
| $M_x$ | 0 | - | - | - | - | - |
| $M_y$ | 0 | 0.3 | 0.6 | 0.8 | 1.1 | 1.3 |

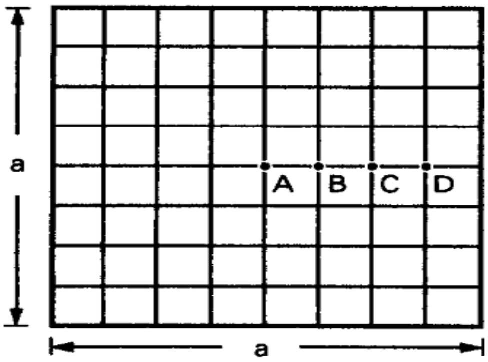


**Figure XIII**

Mesh of the Plate*

*courtesy-Mechanics of laminated composite plates and shells by j.n reddy

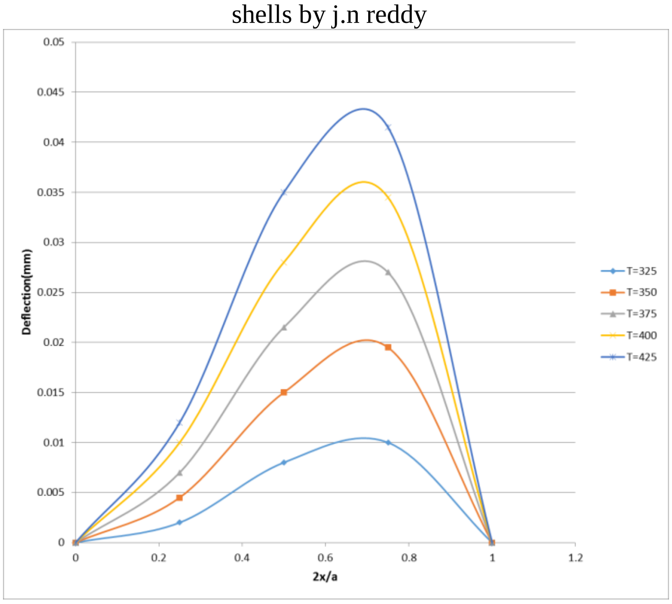


**Figure XIV:** Transverse Deflection Along X Axis For Different Temperature (Simply Supported)

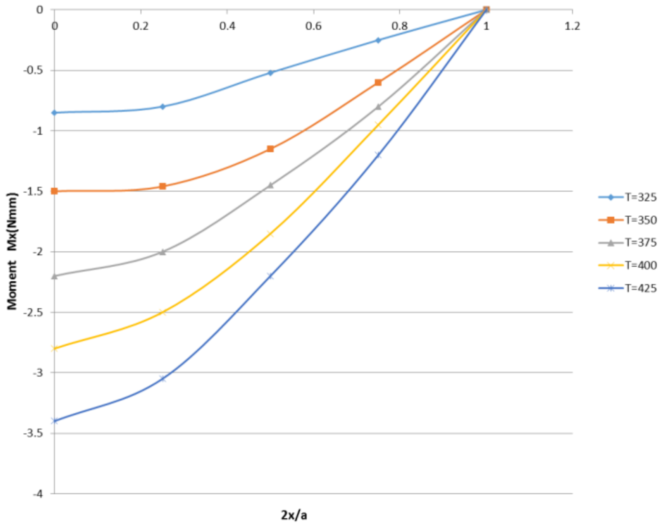


**Figure XV:** Moment $M_X$ for Different Temperature (Simply Supported)

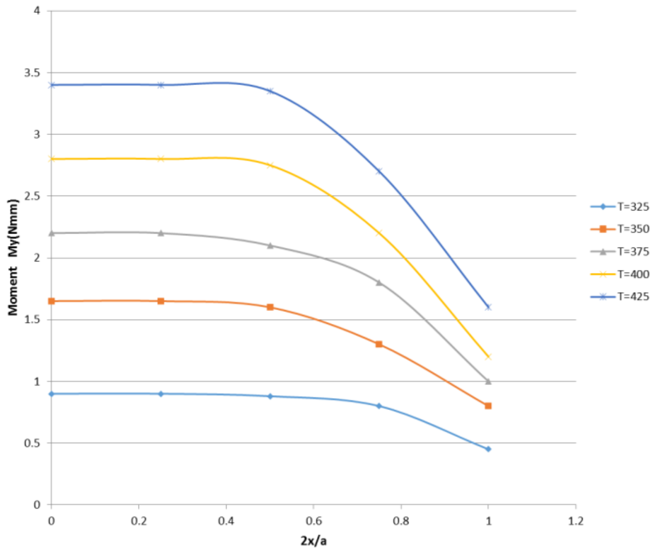


**Figure XVI:** Moment $M_Y$ for Different Temperature (Simply Supported)

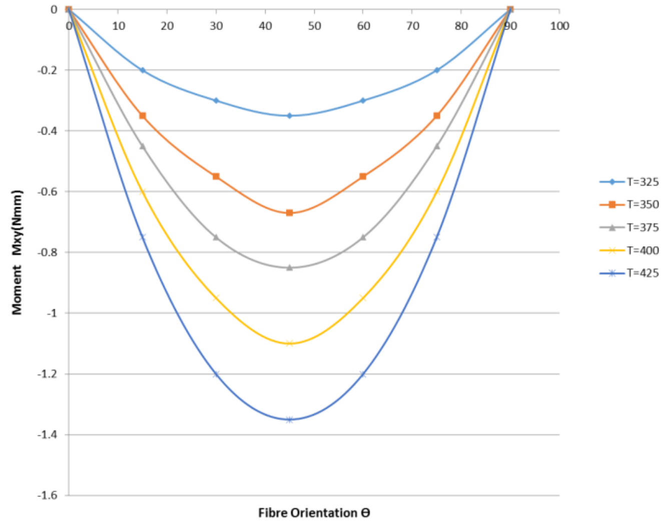


**Figure XVII:** Variation of $M_{XY}$ With Fiber Orientation for Different Temperature (Simply Supported)

## 5. Observations

### A. When Mechanical Load is applied

It is also observed that, the normal stress varies non-linearly across the thickness of the laminate. However, they vary linearly for an individual lamina. The stresses become discontinuous across the thickness of the laminate. This means, there exist different values of stresses at the interfaces of the laminate. The generalized displacements only are continuous across the thickness of the laminate. But, the strains and thus the stresses are not continuous at the boundaries. The stress values are maximum at the top and bottom of the laminate with + ve and – ve signs respectively. The transverse shear stresses are constant throughout the thickness. It is because of the use of a constant in calculating the shear stresses, the shear correction factor.

### B. Effect of Moisture Concentration on

1. *Anti-symmetric cross-ply laminate:* The deflection w and the moments $M_x$, and $M_y$ are plotted along the x-axis for the simply-supported boundary conditions at different moisture concentrations and are shown in Fig VIII, Fig IX and Fig X. The deflection is at the peak near the center of the supported edge. The deflection vanishes along the diagonals. Both the moments $M_x$, and $M_y$ are maximum at the center of the plate having the same value but opposite in sign. The plate edges are found to be not free from moment. In case of clamped boundary conditions, the deflection is zero throughout the laminate and $M_x = -M_x^H$, and $M_y = -M_y^H$, where $M_x^H$ and $M_y^H$ are the non-mechanical moment resultants due to moisture.
2. *Anti-symmetric angle-ply laminate:* For the simply-supported/clamped boundary conditions, $M_x$ and $M_y$ are zero throughout the laminate and $M_{xy} = -M_{xy}^H$, where $M_{xy}^H$ is the non-mechanical moment resultant due to moisture. The moment $M_{xy}$ is plotted as a function of the fiber orientation ϴ at different moisture concentrations and is shown in Fig XI. $M_{xy}$ is maximum when the fiber orientation is at 45$^o$. No deflection is produced due to the uniform moisture concentration.

### C. Effect of Temperature on

1. *Anti-symmetric cross-ply laminate:* The plots of deflection w and moments $M_x$, and $M_y$ along the x-axis for the simply-supported boundary conditions are shown in Fig XIV, Fig XV , Fig XVI. For the clamped boundary conditions, $M_x = -M_x^T$ and $M_y = -M_y^T$, where $M_x^T$ and $M_y^T$ are the non-mechanical moment resultants due to temperature. The different values of $M_x$ and $M_y$ at different temperatures are shown in Table VI. The observations made in above section apply in this case also.
2. *Anti-symmetric angle-ply laminate:* For simply-supported/clamped the boundary conditions, $M_x$ and $M_y$ are zero throughout the laminate and $M_{xy} = -M_{xy}^T$ where $M_{xy}^T$ is the non-mechanical moment resultant due to temperature. Plots of the moment $M_{xy}$ as a function of the fibre orientation ϴ are given in Fig XVII. $M_{xy}$ is at the peak when the fiber orientation is at 45$^0$. In this case too, no deflection is produced due to uniform temperature.

## 6. Conclusions

The conventional finite element formulation has been revised to study the hygrothermal effects on the bending behavior of laminated composite plates. The computer code generated, can handle any general pattern of moisture in addition to the external load. From the results presented in the previous section, some broad conclusions may be made. They are:

1. Deflections and stress resultants increase almost linearly with the uniform increase in moisture concentration and temperature.
2. In case of anti-symmetric cross-ply laminates with simply-supported boundary conditions, subjected to uniform moisture and temperature, deflection vanishes along the diagonals, whereas no deflection is produced for clamped boundary conditions.
3. Uniform moisture and temperature also produces no deflection in case of anti-symmetric angle-ply laminates with simply-supported/clamped boundary conditions.

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