

# On Moderate Fuzzy Analytic Hierarchy Process Pairwise Comparison Model

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**Abstract:** Decisions usually involve the getting the best solution, selecting the suitable experiments, most appropriate judgments, taking the quality results etc., using some techniques. Every decision making can be considered as the choice from the set of alternatives based on a set of criteria. The fuzzy analytic hierarchy process is a multi-criteria decision making and is dealing with decision making problems through pairwise comparisons mode [10]. The weight vectors from this comparison model are obtained by using extent analysis method. This paper concern with an alternate method of finding the weight vectors from the original fuzzy AHP decision model (moderate fuzzy AHP model), that has the same rank as obtained in original fuzzy AHP and ideal fuzzy AHP decision models.

**Keywords:** Multi Criteria Decision Making, Fuzzy AHP, Pairwise Comparison Model, Extent Analysis Method. Fuzzy Ideal AHP, Fuzzy Moderate AHP.

## 1. Introduction

The analytical hierarchy process (AHP) was developed by Saaty in 1971. This process is used to find the weight vectors for decision making problems in uncertain situation from the pair wise comparison model with multiple criteria and alternatives. The function of AHP is to systemize complex and unstructured problems, which it resolves hierarchically from the higher levels to lower levels. Through quantitative judgement, AHP simplifies the decision making processes that relied on intuition to obtain the weight of the alternatives corresponding to the criteria or alternative corresponding subcriteria and sub criteria with respect to main criteria and this provides the sufficient information for decision makers. Alternatives with criteria having greater weight gives the higher weight. The AHP performs problem analysis, which can reduce the risk of mistakes in decision making. However, AHP use cannot overcome the subjectivity, inaccuracy, and fuzziness produced when making decisions. So, by introducing and applying fuzzy set theory and fuzzy operation on AHP, which can ameliorate these failures.

Since basic AHP does not include vagueness for personal judgements, it has been improved by benefiting from fuzzy logic approach. In Fuzzy AHP, the pair wise comparisons of both criteria and the alternatives are performed through the linguistic variables, which are represented by triangular numbers. If the uncertainty (fuzziness) of human decision making is not taken into account, the results from the models can be misleading. Fuzzy theory has been applied in a variety of fields since its introduction. Fuzzy AHP methods is proposed to solve various types of problems. The main theme of these methods is using the concepts of fuzzy set theory and hierarchical structure analysis to present systematic approaches in selecting or justifying alternatives. In this study, the extent analysis method by Chang (1992, 1996) is adopted because the steps of this approach are relatively easier, less time taking and less computational expense than many other fuzzy AHP approaches.

Fuzzy set theory was first introduced by Zadeh in 1965; it emphasizes the fuzziness of human thinking, reasoning, and cognition of surroundings. A number of conventional quantitative analysis methods cannot analyze such things efficiently. Furthermore, fuzzy logic can analyze ambiguity and vagueness of the decision making problem. Fuzzy logic is a method to formalize the human capacity of imprecise or approximate reasoning. Such reasoning represents the human ability reason approximately and judge under uncertainty. In fuzzy logic all truth are partial or approximate. In this sense this reasoning has been termed as interpolative reasoning, where the process of interpolating between the binary extremes of true and false is represented by the ability of fuzzy logic to encapsulate partial truths. The fuzzy set can be defined as follows.

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in U\}$$

Where  $\tilde{A}$  is a fuzzy set.  $\mu_{\tilde{A}}(x)$  is called the membership function.  $U$  is the universe of discourse.  $\mu_{\tilde{A}}(x)$  ranges between 0 and 1. This is called the degree of membership. The fuzzy set can better describe the characteristics of things compared to conventional binary logic. In conventional crisp sets, the value of the membership function can only be 0 or 1.

**A Triangular fuzzy Number** is a special case of fuzzy number. It is defined by a triplet  $\tilde{A} = (a, b, c)$ . This representation is interpreted as membership function  $\mu_{\tilde{A}}: \mathbf{R} \rightarrow [0, 1]$  as follows.

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & \text{if } x \leq a \\ \frac{x-a}{b-a} & \text{if } a \leq x \leq b \\ \frac{c-x}{c-b} & \text{if } b \leq x \leq c \\ 0 & \text{if } x > c \end{cases}$$

**Algebraic Operations:** Let  $\tilde{A} = (a_1, b_1, c_1)$  and  $\tilde{B} = (a_2, b_2, c_2)$  be two triangular fuzzy numbers.

- (i) Addition of Triangular Fuzzy Numbers  $\oplus : \tilde{A} \oplus \tilde{B} = (a_1 + a_2, b_1 + b_2, c_1 + c_2)$
- (ii) Multiplication of Triangular Fuzzy Numbers  $\otimes : \tilde{A} \otimes \tilde{B} = (a_1 a_2, b_1 b_2, c_1 c_2); a_1 \geq 0, a_2 \geq 0$
- (iii) Division of Triangular Fuzzy Number  $\oslash : \tilde{A} \oslash \tilde{B} = \left(\frac{a_1}{c_2}, \frac{b_1}{b_2}, \frac{c_1}{a_2}\right); a_1 > 0, a_2 > 0$
- (iv) Inverse of a Triangular Fuzzy Number:  $\tilde{A}^{-1} = (a_1, b_1, c_1)^{-1} = \left(\frac{1}{c_1}, \frac{1}{b_1}, \frac{1}{a_1}\right); a_1 > 0$

**A Triangular Fuzzy Number Matrix** of order  $n \times m$  is defined as  $A = (\tilde{a}_{ij})_{n \times m}$  where  $\tilde{a}_{ij}$  is a triangular fuzzy number. The two sets,  $X = \{x_1, x_2, x_3, \dots, x_n\}$  as an object set, and  $G = \{u_1, u_2, u_3, \dots, u_m\}$  as a goal set, can be defined in initial stage. According to the principles of Chang's extent analysis, each object is considered correspondingly, and extent analysis for each of the goal,  $g_i$  is executed. It means that it is possible to obtain the values of  $m$  extent analyses that can be demonstrated as  $M_{g_i}^1, M_{g_i}^2, \dots, M_{g_i}^m; i=1, 2, \dots, n$ , where  $M_{g_i}^j (j=1, 2, \dots, m)$  are triangular fuzzy numbers. After identifying initial assumptions, Chang's extent analyses [3], [8], [9] can be examined in four main steps:

**Step 1:** The value of fuzzy synthetic extent with respect to the  $i$ th object is represented as,

$$F_i = \sum_{j=1}^m M_{g_i}^j \otimes \left[ \sum_{i=1}^n \sum_{j=1}^m M_{g_i}^j \right]^{-1}$$

operation of  $m$  extent analysis values can be performed for particular matrix such that  $\sum_{j=1}^m M_{g_i}^j = \left( \sum_{j=1}^m l_j, \sum_{j=1}^m m_j, \sum_{j=1}^m u_j \right)$ . Then,

the fuzzy addition operation of  $M_{g_i}^j (j = 1, 2, \dots, m)$  values such that  $\sum_{i=1}^n \sum_{j=1}^m M_{g_i}^j = \left( \sum_{i=1}^n l_i, \sum_{i=1}^n m_i, \sum_{i=1}^n u_i \right)$  are

performed to obtain  $\left[ \sum_{i=1}^n \sum_{j=1}^m M_{g_i}^j \right]^{-1}$ . At the end of the Step 1, the inverse of the determined vector can be expressed as follows.

$$\left[ \sum_{i=1}^n \sum_{j=1}^m M_{g_i}^j \right]^{-1} = \left( \frac{1}{\sum_{i=1}^n u_i}, \frac{1}{\sum_{i=1}^n m_i}, \frac{1}{\sum_{i=1}^n l_i} \right)$$

**Step 2 :** The degree of possibility of  $M_1=(l_1, m_1, u_1) \geq M_2 = (l_2, m_2, u_2)$  is defined as  $D(M_1 \geq M_2) = \sup_{x \geq y} [\min(\mu_{M_1}(x), \mu_{M_2}(x))]$ ,

When a pair  $(x, y)$  exists such that  $x \geq y$  and  $\mu_{M_1}(x) = \mu_{M_2}(x)$ , then we have  $D(M_1 \geq M_2) = 1$ . Since  $M_1$  and  $M_2$  are convex fuzzy numbers we have that

$$D(M_1 \geq M_2) = 1 \text{ if } m_1 \geq m_2$$

$$D(M_2 \geq M_1) = \text{hgt}(M_1 \cap M_2) = \mu_{M_1}(d)$$

Where  $d$  is the ordinate of the highest intersection point between  $\mu_{M_1}(d)$  and  $\mu_{M_2}(d)$ . Also the above equation can be equivalently expressed as follows.

$$D(M_2 \geq M_1) = \text{hgt}(M_1 \cap M_2) = \mu_{M_1}(d)$$

$$= \begin{cases} 1, & \text{if } m_2 \geq m_1, \\ 0, & \text{if } l_1 \geq u_2, \\ \frac{l_1 - u_2}{(m_2 - u_2) - (m_1 - l_1)} & \text{Otherwise,} \end{cases}$$

**Step 3 :** From obtaining  $k(k=1, 2, \dots, n)$  convex fuzzy numbers, the degree possibility for a  $i$ th convex fuzzy number to be greater than  $k$  convex fuzzy numbers  $M_i (i = 1, 2, \dots, k)$  can be defined as follows.

$$D(F_i \geq F_k) = D(F_i \geq F_1) \text{ and } D(F_i \geq F_2) \dots D(F_i \geq F_k)$$

$$= D(F_i \geq F_1, F_2, F_3, \dots, F_k) \text{ with } i \neq k.$$

$$d'(A_i) = \min [D(F_i \geq F_1, F_2, F_3, \dots, F_k)] \text{ with } i \neq k.$$

$$W' = (d'(A_1), d'(A_2), \dots, d'(A_n))^T$$

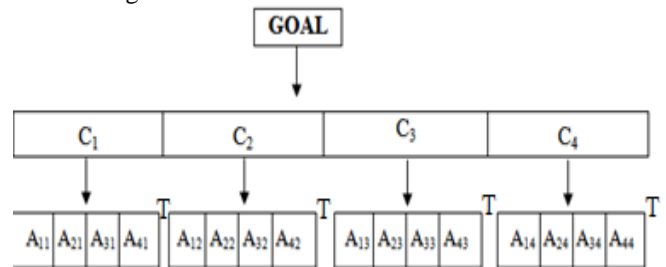
where  $A_i (i = 1, 2, \dots, n)$  are  $n$  elements.

**Step 4:** Via normalization, the normalized weight vectors are  $W = (d(A_1), d(A_2), \dots, d(A_n))^T$ , where  $W$  is a nonfuzzy number that gives weight vectors of an attribute or an alternative over other. Thus we get the original fuzzy AHP decision model with weight vector  $W$ .

**Step 5:** Aggregate the relative weights of decision elements to obtain an overall rating for the alternatives. Finally the alternative with highest weight is chosen as the best alternative.

## 2. Model of the Problem

We define the 4 criteria ( $C_1, C_2, C_3, C_4$ ) and 4 alternatives ( $A_1, A_2, A_3, A_4$ ) in order to obtain the best alternative and criterion figure 1.



**Figure 1:** Selection of Best

Construct the Pair wise comparison model for each criterion  $C_{ij}$  represents the relative weight of the criterion  $C_i$  compared to  $C_j$  as follows

**Pairwise comparison for the Criteria**

Alt. /crit.	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	C <sub>4</sub>	
C <sub>1</sub>	C <sub>11</sub>	C <sub>12</sub>	C <sub>13</sub>	C <sub>14</sub>	C <sub>1j</sub> = (l <sub>1j</sub> , m <sub>1j</sub> , u <sub>1j</sub> ): j = 1,2,3,4 with l <sub>11</sub> = m <sub>11</sub> = u <sub>11</sub> = 1
C <sub>2</sub>	C <sub>21</sub>	C <sub>22</sub>	C <sub>23</sub>	C <sub>24</sub>	C <sub>2j</sub> = (l <sub>2j</sub> , m <sub>2j</sub> , u <sub>2j</sub> ): j = 2,3,4 With l <sub>22</sub> = m <sub>22</sub> = u <sub>22</sub> = 1
C <sub>3</sub>	C <sub>31</sub>	C <sub>32</sub>	C <sub>33</sub>	C <sub>34</sub>	C <sub>3j</sub> = (l <sub>3j</sub> , m <sub>3j</sub> , u <sub>3j</sub> ): j = 3,4 With l <sub>33</sub> = m <sub>33</sub> = u <sub>33</sub> = 1
C <sub>4</sub>	C <sub>41</sub>	C <sub>42</sub>	C <sub>43</sub>	C <sub>44</sub>	C <sub>4j</sub> = (l <sub>4j</sub> , m <sub>4j</sub> , u <sub>4j</sub> ): j = 4 With l <sub>44</sub> = m <sub>44</sub> = u <sub>44</sub> = 1

$$C_{i1} = \left( \frac{1}{u_{1i}}, \frac{1}{m_{1i}}, \frac{1}{l_{1i}} \right) \quad C_{i2} = \left( \frac{1}{u_{2i}}, \frac{1}{m_{2i}}, \frac{1}{l_{2i}} \right) \quad C_{i3} = \left( \frac{1}{u_{3i}}, \frac{1}{m_{3i}}, \frac{1}{l_{3i}} \right)$$

i = 2, 3, 4 i = 3, 4 i = 4.

Applying all steps of Chang's extent analysis method in the above model, we have the weight vectors of the Criteria C<sub>1</sub>, C<sub>2</sub>, C<sub>3</sub> and C<sub>4</sub> are obtained as W<sub>c</sub>=(d(C<sub>1</sub>), d(C<sub>2</sub>), d(C<sub>3</sub>), d(C<sub>4</sub>)).

Similarly, Construct the pair wise comparison model for the alternatives A<sub>11</sub>, A<sub>21</sub>, A<sub>31</sub>, A<sub>41</sub>, with respect to the criterion C<sub>1</sub> and applying all the steps of Chang's extent analysis method in the model. Obtain the weight vectors corresponding to A<sub>11</sub>, A<sub>21</sub>, A<sub>31</sub>, A<sub>41</sub> respectively

$$d(A_{11}), d(A_{21}), d(A_{31}), d(A_{41}) \text{ as}$$

$$w_{C_1} = (d(A_{11}), d(A_{21}), d(A_{31}), d(A_{41}))^T$$

Similarly, Construct the pair wise comparison model for the alternatives A<sub>12</sub>, A<sub>22</sub>, A<sub>32</sub>, A<sub>42</sub>, with respect to the criterion C<sub>2</sub> and applying all the steps of Chang's extent analysis method in the model. Obtain the weight vectors corresponding to A<sub>12</sub>, A<sub>22</sub>, A<sub>32</sub>, A<sub>42</sub> respectively

$$d(A_{12}), d(A_{22}), d(A_{32}), d(A_{42}) \text{ as}$$

$$w_{C_2} = (d(A_{12}), d(A_{22}), d(A_{32}), d(A_{42}))^T$$

Similarly, Construct the pair wise comparison model for the alternatives A<sub>13</sub>, A<sub>23</sub>, A<sub>33</sub>, A<sub>43</sub>, with respect to the criterion C<sub>3</sub> and applying all the steps of Chang's extent analysis method in the model. Obtain the weight vectors corresponding to A<sub>13</sub>, A<sub>23</sub>, A<sub>33</sub>, A<sub>43</sub> respectively

$$d(A_{13}), d(A_{23}), d(A_{33}), d(A_{43}) \text{ as}$$

$$w_{C_3} = (d(A_{13}), d(A_{23}), d(A_{33}), d(A_{43}))^T$$

Similarly, Construct the pair wise comparison matrix for the alternatives A<sub>14</sub>, A<sub>24</sub>, A<sub>34</sub>, A<sub>44</sub>, with respect to the criterion C<sub>4</sub> and applying all the steps of Chang's extent analysis method in the model. Obtain the weight vectors corresponding to A<sub>14</sub>, A<sub>24</sub>, A<sub>34</sub>, A<sub>44</sub> respectively

$$d(A_{14}), d(A_{24}), d(A_{34}), d(A_{44}) \text{ as}$$

$$w_{C_4} = (d(A_{14}), d(A_{24}), d(A_{34}), d(A_{44}))^T$$

Thus we get the original fuzzy AHP decision model and their weight vectors, using above weight vectors of Criteria and alternatives

**Fuzzy AHP model**

Alternative /criterion	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	C <sub>4</sub>	Final Weight Vector
Weight Vectors of Criteria	d(C <sub>1</sub> )	d(C <sub>2</sub> )	d(C <sub>3</sub> )	d(C <sub>4</sub> )	A <sup>2</sup> <sub>AHP</sub>
A <sub>1</sub>	d(A <sub>11</sub> )	d(A <sub>12</sub> )	d(A <sub>13</sub> )	d(A <sub>14</sub> )	A <sup>1</sup> <sub>AHP</sub>
A <sub>2</sub>	d(A <sub>21</sub> )	d(A <sub>22</sub> )	d(A <sub>23</sub> )	d(A <sub>24</sub> )	A <sup>2</sup> <sub>AHP</sub>
A <sub>3</sub>	d(A <sub>31</sub> )	d(A <sub>32</sub> )	d(A <sub>33</sub> )	d(A <sub>34</sub> )	A <sup>3</sup> <sub>AHP</sub>
A <sub>4</sub>	d(A <sub>41</sub> )	d(A <sub>42</sub> )	d(A <sub>43</sub> )	d(A <sub>44</sub> )	A <sup>4</sup> <sub>AHP</sub>

Ideal fuzzy AHP decision model using original fuzzy AHP decision model and their weight vector are obtained as follows.

**Ideal AHP model**

Alternative /criterion	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	C <sub>4</sub>	Final Weight Vector
Weight Vectors of Criteria	d(C <sub>1</sub> )	d(C <sub>2</sub> )	d(C <sub>3</sub> )	d(C <sub>4</sub> )	
A <sub>1</sub>	d(IA <sub>11</sub> )	d(IA <sub>12</sub> )	d(IA <sub>13</sub> )	d(IA <sub>14</sub> )	IA <sub>1</sub>
A <sub>2</sub>	d(IA <sub>21</sub> )	d(IA <sub>22</sub> )	d(IA <sub>23</sub> )	d(IA <sub>24</sub> )	IA <sub>2</sub>
A <sub>3</sub>	d(IA <sub>31</sub> )	d(IA <sub>32</sub> )	d(IA <sub>33</sub> )	d(IA <sub>34</sub> )	IA <sub>3</sub>
A <sub>4</sub>	d(IA <sub>41</sub> )	d(IA <sub>42</sub> )	d(IA <sub>43</sub> )	d(IA <sub>44</sub> )	IA <sub>4</sub>

After normalization, we have the ranking the alternatives.

It can also be extended to find the final alternative weight vectors for each alternative from the original fuzzy AHP decision model. It can be obtained from the following ways.

$$MA_i = \sum_{j=1}^4 d(C_j) [d(C_j) + d(A_{ij})] \text{ for all}$$

Thus the moderate fuzzy AHP [7] decision matrix is obtained as follows.

**Moderate AHP Model**

	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	C <sub>4</sub>	Final Weight Vector
Weight Vectors of Criteria	d(C <sub>1</sub> )	d(C <sub>2</sub> )	d(C <sub>3</sub> )	d(C <sub>4</sub> )	
A <sub>1</sub>	d(MA <sub>11</sub> )	d(MA <sub>12</sub> )	d(MA <sub>13</sub> )	d(MA <sub>14</sub> )	MA <sub>1</sub>
A <sub>2</sub>	d(MA <sub>21</sub> )	d(MA <sub>22</sub> )	d(MA <sub>23</sub> )	d(MA <sub>24</sub> )	MA <sub>2</sub>
A <sub>3</sub>	d(MA <sub>31</sub> )	d(MA <sub>32</sub> )	d(MA <sub>33</sub> )	d(MA <sub>34</sub> )	MA <sub>3</sub>
A <sub>4</sub>	d(MA <sub>41</sub> )	d(MA <sub>42</sub> )	d(MA <sub>43</sub> )	d(MA <sub>44</sub> )	MA <sub>4</sub>

After normalization, we have the ranking the alternatives. Finally we have the same ranking for original fuzzy AHP decision model, Ideal fuzzy AHP decision model and moderate fuzzy AHP decision model, even though different the value of the final alternative weight vectors of respective alternatives.

### 3. Numerical Example

Decision makers determine goal, Criteria and alternative of the problem in a hierarchical form. This hierarchy has to give the all details of the information on the structure in order to give lack less of the problem. Decision makers are required to compare each factor in the hierarchy. The evaluation of the fuzzy scale used by the decision makes shown in the table 1.

**Table 1:** Fuzzy AHP scale

S. No.	Definition	Triangular Fuzzy Number	Reciprocal Fuzzy Number
1.	Equally importance	(1,1,1)	(1,1,1)
2.	Weakly importance	(1,1,3)	(1/3, 1,1)
3.	Moderately importance	(1,3,3)	(1/3, 1/3,1)
4.	Strongly importance	(1,3,5)	(1/5, 1/3, 1)
5.	Very strongly importance	(3,5,7)	(1/7, 1/5, 1/3)
6.	Extremely importance	(5,7,9)	(1/9, 1/7, 1/5)

**Pairwise comparison of Criteria**

Alternative /criterion	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	C <sub>4</sub>
C <sub>1</sub>	(1,1,1)	(1,1,3)	(1,3,3)	(3,5,7)
C <sub>2</sub>	( $\frac{1}{3}, 1, 1$ )	(1,1,1)	(1,3,5)	(3,5,7)
C <sub>3</sub>	( $\frac{1}{5}, \frac{1}{3}, 1$ )	( $\frac{1}{5}, \frac{1}{3}, 1$ )	(1,1,1)	(1,3,5)
C <sub>4</sub>	( $\frac{1}{7}, \frac{1}{5}, \frac{1}{3}$ )	( $\frac{1}{7}, \frac{1}{5}, \frac{1}{3}$ )	( $\frac{1}{5}, \frac{1}{3}, 1$ )	(1,1,1)

The normalized weight vector for criteria are calculated as  $w_c = (0.3704, 0.3704, 0.2386, 0.0206)$

**Pairwise comparison matrix for alternatives with respect to C<sub>1</sub>**

C <sub>1</sub>	A <sub>11</sub>	A <sub>21</sub>	A <sub>31</sub>	A <sub>41</sub>
A <sub>11</sub>	(1,1,1)	(1,1,3)	( $\frac{1}{5}, \frac{1}{3}, 1$ )	(3,5,7)
A <sub>21</sub>	( $\frac{1}{3}, 1, 1$ )	(1,1,1)	(1,1,3)	(5,7,9)
A <sub>31</sub>	(1, 3, 5)	( $\frac{1}{3}, 1, 1$ )	(1,1,1)	( $\frac{1}{9}, \frac{1}{7}, \frac{1}{5}$ )
A <sub>41</sub>	( $\frac{1}{7}, \frac{1}{5}, \frac{1}{3}$ )	( $\frac{1}{9}, \frac{1}{7}, \frac{1}{5}$ )	(5,7,9)	(1,1,1)

The normalized weight vectors with respect to C<sub>1</sub> are calculated as  $w_{C_1} = (0.2565, 0.3124, 0.1627, 0.2684)^T$

**Pairwise comparison model for alternatives with respect to C<sub>2</sub>**

2	A <sub>12</sub>	A <sub>22</sub>	A <sub>32</sub>	A <sub>42</sub>
A <sub>12</sub>	(1,1,1)	(1,1,3)	(3,5,7)	( $\frac{1}{5}, \frac{1}{3}, 1$ )
A <sub>22</sub>	( $\frac{1}{3}, 1, 1$ )	(1,1,1)	(1,1,3)	( $\frac{1}{9}, \frac{1}{7}, \frac{1}{5}$ )
A <sub>32</sub>	( $\frac{1}{7}, \frac{1}{5}, \frac{1}{3}$ )	( $\frac{1}{3}, 1, 1$ )	(1,1,1)	(1,3,5)

**Ideal mode fuzzy AHP decision model**

Alternative /criterion	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	C <sub>4</sub>	Final weight Vector	Normali zation	Ranking
Crit Weight	0.3704	0.3704	0.2386	0.0206			
A <sub>1</sub>	0.8211	0.7767	0.6306	1.0000	0.7629	0.2722	2
A <sub>2</sub>	1.0000	0.2870	0.7818	0.5045	0.6736	0.2404	3
A <sub>3</sub>	0.5208	0.5198	0.1543	0.7981	0.4386	0.1565	4
A <sub>4</sub>	0.8592	1.0000	1.0000	0.0000	0.9272	0.3309	1

**Moderate fuzzy AHP decision model**

Alternative /criterion	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	C <sub>4</sub>	Final weight Vector	Normali zation	Ranking
Crit Weight	0.3704	0.3704	0.2386	0.0206			
A <sub>1</sub>	0.2322	0.2485	0.1156	0.0094	0.6057	0.2603	2
A <sub>2</sub>	0.2529	0.1785	0.1296	0.0049	0.5659	0.2432	3
A <sub>3</sub>	0.1975	0.2117	0.0713	0.0076	0.4881	0.2098	4
A <sub>4</sub>	0.2366	0.2805	0.1499	0.0000	0.6670	0.2866	1

A <sub>42</sub>	(1, 3, 5)	(5,7,9)	( $\frac{1}{5}, \frac{1}{3}, 1$ )	(1,1,1)
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The normalized  $w_{C_2} = (0.3005, 0.1114, 0.2011, 0.3869)^T$

**Pairwise comparison model for alternatives with respect to C<sub>3</sub>**

C <sub>3</sub>	A <sub>13</sub>	A <sub>23</sub>	A <sub>33</sub>	A <sub>43</sub>
A <sub>13</sub>	(1,1,1)	(1,1,3)	(1,3,5)	( $\frac{1}{9}, \frac{1}{7}, \frac{1}{5}$ )
A <sub>23</sub>	( $\frac{1}{3}, 1, 1$ )	(1,1,1)	(1,3,3)	(1,3,5)
A <sub>33</sub>	( $\frac{1}{5}, \frac{1}{3}, 1$ )	( $\frac{1}{3}, \frac{1}{3}, 1$ )	(1,1,1)	( $\frac{1}{3}, \frac{1}{3}, 1$ )
A <sub>43</sub>	(5,7,9)	( $\frac{1}{5}, \frac{1}{3}, 1$ )	(1,3,3)	(1,1,1)

The normalized weight vectors with respect to C<sub>3</sub> are calculated as  $w_{C_3} = (0.2457, 0.3046, 0.0601, 0.3896)^T$

**Pairwise comparison model for alternatives with respect to C<sub>4</sub>**

C <sub>4</sub>	A <sub>14</sub>	A <sub>24</sub>	A <sub>34</sub>	A <sub>44</sub>
A <sub>14</sub>	(1,1,1)	(1,1,3)	(3,5,7)	(3,5,7)
A <sub>24</sub>	( $\frac{1}{3}, 1, 1$ )	(1,1,1)	(1,1,3)	(1,3,3)
A <sub>34</sub>	( $\frac{1}{7}, \frac{1}{5}, \frac{1}{3}$ )	( $\frac{1}{3}, 1, 1$ )	(1,1,1)	(5,7,9)
A <sub>44</sub>	( $\frac{1}{7}, \frac{1}{5}, \frac{1}{3}$ )	( $\frac{1}{3}, \frac{1}{3}, 1$ )	( $\frac{1}{9}, \frac{1}{7}, \frac{1}{5}$ )	(1,1,1)

The normalized weight vectors with respect to C<sub>4</sub> are calculated as

$$w_{C_4} = (0.4343, 0.2191, 0.3466, 0.0000)^T$$

From weight vectors of criteria and alternatives, we have fuzzy AHP decision models as follows.

**Thus original fuzzy AHP decision model**

Alternative /criterion	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	C <sub>4</sub>	Final weight Vector	Ranking
Crit Weight	0.3704	0.3704	0.2386	0.0206		
A <sub>1</sub>	0.2565	0.3005	0.2457	0.4343	0.2738	2
A <sub>2</sub>	0.3124	0.1114	0.3046	0.2191	0.2342	3
A <sub>3</sub>	0.1627	0.2011	0.0601	0.3466	0.1562	4
A <sub>4</sub>	0.2684	0.3869	0.3896	0.0000	0.3357	1

Therefore, the best selection is  $A_4$  followed by  $A_1$ ,  $A_1$  is followed by  $A_2$  and  $A_2$  is followed by  $A_3$ . Finally we observe that the original fuzzy AHP, the ideal fuzzy AHP and the moderate fuzzy AHP have the same ranking for the said 4 alternatives, even though they assigned different final weight vectors for these alternatives.

#### 4. Conclusion

The fuzzy AHP is used for ranking with weight vectors of pairwise comparison matrices. It provides an effective solution for solving MCDM problem. We can involve any relative importance of criteria and that of alternatives in the moderate fuzzy AHP. Also moderate fuzzy AHP allows for a sensitivity analysis in term of the relative priorities, by adjusting the ranking values. Application of the moderate fuzzy AHP of the MCDM can be discussed in further research proposals. The numerical problem shows the proposed fuzzy analysis and its applicability in providing a valuable decision support.

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