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Behaviour of Time Displacement Field and String Tension Density for Bianchi Type-V Model with Dark Energy

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Abstract: ***In the present work we have studied the behaviour of time displacement field and string tension density using $p=-\rho$ in Bianchi-v space time. For solving the Einstein's field equation we choose the scale factor $a=\left(t^n e^t\right)^{\frac{1}{2}}$ which yields a time dependent deceleration parameter representing a model which generates a transition of the universe from the early decelerating phase to the recent accelerating phase. Some physical parameters are also discussed. In this paper we have examined the time displacement field and string tension density depends on EOS parameter.***

Keywords: Time displacement field, String tension density, Dark energy

1. Introduction

It is commonly believed by the cosmological community that a kind of repulsive force which acts as antigravity is responsible for gearing up the universe some 7 billion years ago. This hitherto unknown exotic physical entity is therefore termed as dark energy. The nature of dark energy lies in, the equation of state parameter which is nothing but the ratio of fluid pressure and matter energy density of dark energy, i.e. This parameter has been selected in different ways in different models. Recent observations of type Ia supernovae suggest that the expansion of the universe is accelerating and two thirds of the total energy density exist in a dark energy component with negative pressure [1,2]. It is not even known what is the current value of the dark energy effective equation of state parameter which lies close to -1, it could be equal to -1, a little bit upper than -1 or less than -1. The total result in 2009,obtained after a combination of cosmology datasets coming from CMB anisotropies luminosity distances of high red shift type Ia supernovae and galaxy clustering, constrain the dark energy EOS to at confidence level[3,4].

Inspired by Einstein's geometrized gravitation Weyl[5] try to unify gravity and electromagnetism. But this theory was not accepted as it was based on non integrability of length transfer. Then Lyra[6] proposed a new modification of Riemannian geometry by introducing gauge function for removing the non integrability of length transfer. So Einstein's field equation based on Lyra geometry may be written as

$$R^i{}_j-\frac{1}{2}g^i{}_jR+\frac{3}{2}\phi_i\phi_j-\frac{3}{4}g^i{}_j\phi_k\phi^k=-8\pi GT^i{}_j$$

Some very recent works done on Lyra geometry are given in [7,8], viz., Rahaman et al.[7] have done investigation in cosmology within the framework of Lyra geometry. Rahaman et al[8] have studied string in five dimensional space time based on Lyra geometry and in one model they have shown that the gauge function is large in the beginning but decreases with the evolution of the model.

In the study of physical situation at the early stages of the universe string theory is very helpful for researchers. Basically it is believed that after the big bang, the universe may have undergone a series of phase transition as its temperature was lowered down below some critical temperature. At the beginning of the formation of the universe during phase transition it is known that the symmetry of the universe is broken spontaneously. Cosmic strings create a considerable interest as these act as gravitational lenses and give rise to density perturbations leading to the formation of galaxies.

2. Bianchi-v Model and Field Equation

The line element for the spatially homogeneous and anisotropic Bianchi-v space time is given by

$$ds^2=-dt^2+A^2dx^2+e^{2\alpha x}\left(B^2dy^2+C^2dz^2\right) \tag{1}$$

Where A, B and C are scale factor in different directions and α is a constant.

We assume $a=\left(ABC\right)^{\frac{1}{3}}$ as the average scale factor so that the Hubble parameter

$$H=\frac{\dot{a}}{a} \tag{2}$$

Einstein's field equation based on Lyra geometry

$$R^i{}_j-\frac{1}{2}g^i{}_jR+\frac{3}{2}\phi_i\phi_j-\frac{3}{4}g^i{}_j\phi_k\phi^k=-8\pi GT^{(m)i}{}_j-T^{(de)i}{}_j \tag{3}$$

The energy momentum tensor $T^i{}_j$ for a cloud of massive strings and the distribution of perfect fluid is taken as

$$T^{(m)i}{}_j=(\rho+p)v^iv_j-pg^i{}_j-\lambda a^ia_j \tag{4}$$

$$T^{(de)i}{}_j=diag\left(-\rho,p,p,p\right) \tag{5}$$

Where $\rho^{(m)}$ and $p^{(m)}$ are respectively the energy density and pressure of the perfect fluid component or ordinary matter while $\omega^{(m)} = \frac{p}{\rho}$. Similarly $\rho^{(de)}$ and $p^{(de)}$ are respectively the energy density and pressure of the DE component while $\omega^{(de)} = \frac{p^{(de)}}{\rho^{(de)}}$.

For equation (4),p is isotropic pressure; ρ is the proper energy density for a cloud strings with particle attached to them; λ is the string tension density; $v^i = (0,0,0,1)$ is the four velocity of the particles and x^i is a unit space-like vector representing the direction of string. The vector v^i and x^i satisfies the conditions

$$v_i v^i = -x_i x^i = -1 \tag{6}$$

$$x^i = \left(A^{-1},0,0,0\right) \tag{7}$$

If the particle density of the configuration denoted by ρ_p ,then

$$\rho_p = \rho - \lambda \tag{8}$$

The time-like displacement vector ϕ_i is defined as

$$\phi_i = (\beta,0,0,0) \tag{9}$$

The Einstein's field equation for the line element lead to

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{\alpha^2}{A^2} = -p^{(m)} - \frac{3}{4}\beta^2 + \lambda - \omega^{(de)}\rho^{(de)} \tag{10}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{\alpha^2}{A^2} = -p^{(m)} - \frac{3}{4}\beta^2 - \omega^{(de)}\rho^{(de)} \tag{11}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{\alpha^2}{A^2} = -p^{(m)} - \frac{3}{4}\beta^2 - \omega^{(de)}\rho^{(de)} \tag{12}$$

$$\frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} + \frac{\dot{A}\dot{B}}{AB} - \frac{3\alpha^2}{A^2} = \rho^{(m)} + \frac{3}{4}\beta^2 + \rho^{(de)} \tag{13}$$

$$\frac{2\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0 \tag{14}$$

The energy conservation equation gives

$$\zeta\dot{\rho} + \frac{3}{2}\beta\dot{\beta} + \left[\zeta(\rho+p) + \frac{3}{2}\beta^2\right]\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = 0 \tag{15}$$

Again energy conservation equation $T^i{}_{j;j} = 0$ gives

$$\dot{\rho} + (\rho+p)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) - \lambda\frac{\dot{A}}{A} = 0 \tag{16}$$

3. Solution of the Field Equation:

From equation (12), we get

$$A^2 = BC \tag{17}$$

Using (15), substracting (9) from (10), we get

$$\frac{B}{C} = k_1 \exp\left(\int \frac{k}{ABC}dt\right) \tag{18}$$

Equation (8)-(12) are five independent equation in 9 unknowns A, B, C, $\rho^{(m)}$, $p^{(m)}$, $\rho^{(de)}$, $p^{(de)}$, λ and β.

We need four extra conditions. So we assume

$$p = -\rho \tag{19}$$

And

$$a = \left(t^n e^t\right)^{\frac{1}{2}} \tag{20}$$

Where m and n are positive constants.

The energy conservation equation $T^{(m)ij}{}_{ij} = 0$ of the perfect fluid gives

$$\dot{\rho}^{(m)} + 3\rho^{(m)}(1+\omega)H = 0 \tag{21}$$

Where ω time dependent

The energy conservation equation $T^{(de)ij}{}_{ij} = 0$ of the DE component lead to

$$\dot{\rho}^{(de)} + 3\rho^{(de)}(1+\omega)H = 0 \tag{22}$$

Now the spatial volume V of the model read as

$$V = a^3 = \left(t^n e^t\right)^{\frac{3}{2}} \tag{23}$$

Equation (15),(18) and (21) lead to

$$A(t) = \left(t^n e^t\right)^{\frac{1}{2}} \tag{24}$$

Inserting

$$B = \left(t^n e^t\right)^{\frac{1}{2}}\left\{k_1 \exp\left(\int \frac{k}{ABC}dt\right)\right\} \tag{25}$$

$$C = \left(t^n e^t\right)^{\frac{1}{m}}\left\{k_1 \exp\left(-\int \frac{k}{ABC}dt\right)\right\} \tag{26}$$

4. Some Physical and Geometrical Properties

The time-displacement field β ,Pressure p ,density ρ , the string tensor density (λ), particle density ρ_p, deceleration parameter q are given by

$$\beta^2 = \frac{2}{3}\left[\frac{2}{m^2}\left(\frac{n}{t}+1\right)^2 + \frac{k}{2m}\left(\frac{n}{t}+1\right)\left(a^{-3} - 2a^{-1}\right) - \frac{k^2}{4}\left(a^{-4} + a^{-6}\right) - 2\alpha^2 a^{-2} - \frac{2n(n-m)}{m^2}t^{-2}\right] + \frac{2}{3} \tag{27}$$

$$\left[-\frac{4n}{m^2}t^{-1} - \frac{1}{m^2} - \frac{3nk}{m}t^{-1}a^{-3} - \frac{3k}{m}a^{-3} - (\omega+1)a^{-3(\omega+1)}\right]$$

$$\rho^{(de)} = a^{-3(1+\omega)} \tag{28}$$

$$\rho^{(m)} = a^{-3(1+\omega)} \tag{29}$$

$$\lambda=\left[\begin{array}{l}\frac{k^2}{8}a^{-4}\left(a^{-2}-1\right)+\frac{nk}{m}t^{-1}a^{-3}-\frac{nk}{2m}t^{-1}a^{-1}+\frac{k}{m}a^{-3}+\frac{nk}{2m}t^{-1}a\ +\frac{2}{m^2}\left(\frac{n}{t}+1\right)^2\\ +\frac{k}{2m}\left(\frac{n}{t}+1\right)\left(a^{-3}-a^{-1}\right)-2\alpha^2a^{-2}+\frac{k}{4m}\left(\frac{n}{t}+1\right)\left(a^{-3}-2a^{-1}\right)+\frac{n(n-m)}{m^2}t^{-2}\\ +\frac{2n}{m^2}t^{-1}+\frac{1}{2}(\omega-3)a^{-3(1+\omega)}\end{array}\right]+\frac{1}{2m^2} \tag{30}$$

We define the deceleration parameter q as usual. I,e

$$q=-\frac{a\ddot{a}}{\dot{a}^2}=-\frac{\ddot{a}}{aH^2} \tag{31}$$

Using the average scale factor

$$q=-1+\frac{2n}{(n+t)^2} \tag{32}$$

From equ (29), we observed that $q>0$ for $t<\sqrt{2n}-n$ and $q<0$ for $t>\sqrt{2n}-n$.It is observed that for $0<n<2$,our model is evolving from deceleration phase to acceleration phase. Also recent observations of SNe Ia, expose that the present universe is accelerating and the value of DP lies to some place in the range $-1<q<0$. It follows that in our derived model one can choose the value of DP consistent with the observations. Figure 3 graphs the deceleration parameter (q) versus time which gives the behaviour of q from decelerating to accelerating phase for different values of n. The expressions for physical parameters such as the directional Hubble parameter, Hubble parameter(H), scalar of expansion (θ), shear scalar(σ), spatial volume V and the anisotropy parameter are respectively given by

$$H_x=\frac{1}{2}\left(\frac{n}{t}+1\right) \tag{33}$$

$$H_y=\frac{1}{2}\left(\frac{n}{t}+1\right)+k(t^ne^t)^{\frac{-3}{2}} \tag{34}$$

$$H_z=\frac{1}{2}\left(\frac{n}{t}+1\right)-k(t^ne^t)^{\frac{-3}{2}} \tag{35}$$

$$\theta=3H=\frac{3}{2}\left(\frac{n}{t}+1\right) \tag{36}$$

$$\sigma^2=k^2(t^ne^t)^{-3} \tag{37}$$

$$V=(t^ne^t)^{\frac{3}{2}}\exp(2\alpha x) \tag{38}$$

$$A_m=\frac{8k^2}{3}\left(\frac{n}{3}+1\right)^{-2}(t^ne^t)^{-3} \tag{39}$$

It is examined that the spatial volume vanishes at $t=0$.

And also observed that physical parameters θ,σ, and H diverge. Since directional scale factor vanish at initial time, it is a Point Type Singularoty (MacCallum 1971).

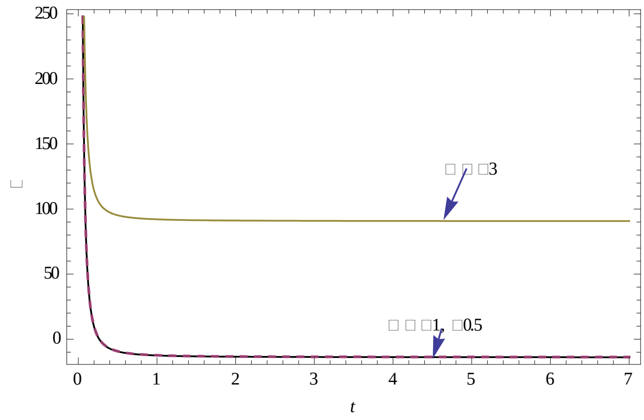

Figure 1

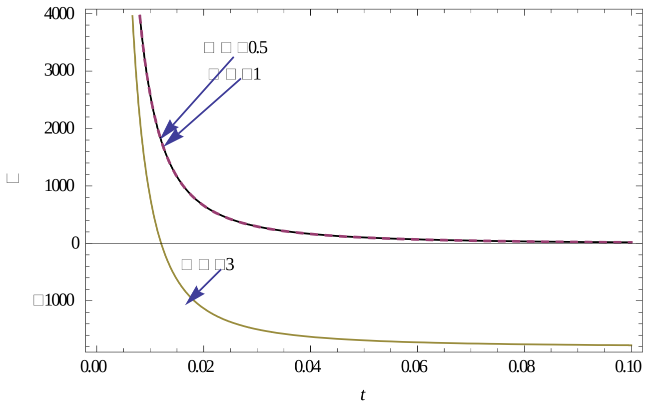

Figure 2

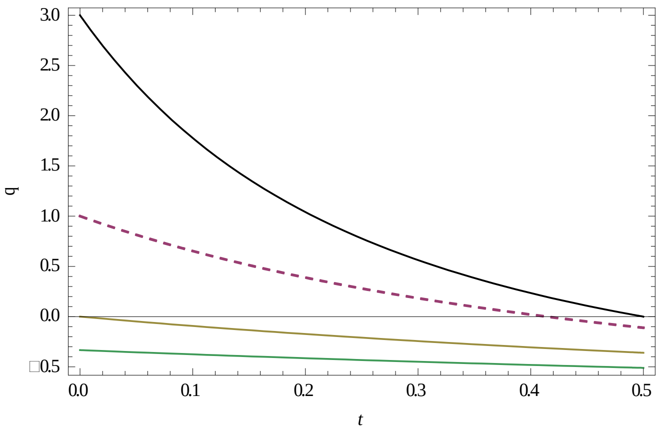

Figure 3

5. Conclusion

In this paper we have discussed about the behaviour of time displacement field and string tension density in the presence of dark energy. We have studied a spatially homogeneous and anisotropic Bianchi type-v space time filled with perfect fluid with Lyra geometry and dark energy. The field equations have been solved with suitable physical assumptions. Kumar and Yadhab [9] have solved the field equations by considering the constant DP whereas we have considered time dependent DP. Here we have examined for a universe which was decelerating in past and accelerating at the present time, the DP must show signature flipping and so there is no scope for a constant DP. The main features of the model are as follows:

- The present DE model has a transition of the universe from the early deceleration phase to the recent acceleration phase (see, Figure) which is in good agreement with recent observations [11].
- Our special choice of scale factor yields a time dependent deceleration parameter which represent a model of the universe which evolves from decelerating phase to an accelerating phase whereas in Yadhab[10], Kumar and Yadhab[9] only the evolution takes place either in an accelerating or decelerating phase.
- For different choice of n,we can generate a class of DE models in Bianchi type-v space-time. It is observed that such DE models are also in good harmony with current observations.
- From Figure-1 it is observed that the behaviour of time displacement field depends on EOS parameter. Fromaure-2 we have also examined that string tension density depends on EOS parameter.

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