Remarks on δg^*- Closed Sets in Digital Line

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Abstract: ***We establish the connection between two fields of Topology that until now seemed to be far away from each other, namely digital topology and the theory of generalized closed sets. In this paper we study delta-generalized star closed sets and some allied concepts applied to digital topology.***

Keywords: δg^*-closed sets, δg^*-kernel and δg-closed

1. Introduction

General topology has applications in the theory of image processing by exhibiting algorithms, which apply current knowledge of digital spaces. The major building block of the digital n-space is the digital line or the Khalimsky line [8]. In this section we show that the digital line $(Z, \mathcal{K})$ is a ${}_{\delta g^*}T_\delta$-space, a ${}_{\delta g}T_{\delta g^*}$-space, a ${}_{\delta \hat{g}}T_{\delta g^*}$-space, ${}_{w\delta g^*}T_\delta$-space, ${}_{w\delta g^*}T_{\delta g^*}$-space and ${}_{\delta g}T_{w\delta g^*}$-spaces. Characterizations of δg^*-open sets in the digital line are obtained.

2. Preliminaries

Throughout this paper (X, τ) (or simple X) represents topological space on which no separation axioms are assumed unless otherwise mentioned. For a subset A of X, cl(A), int(A) and A^c denote the closure of A, the interior of A and the complement of A respectively. Let us recall the following definitions, which are useful in the sequel.

2.1 Definition

Let (X, τ) be a topological space. A subset A of (X, τ) is called

(i) regular closed set [13] if A = cl(int(A))
(ii) semi-closed set [10] if $\text{int(cl(A))} \subseteq A$

The complements of the above mentioned sets are called regular open and semi-open.

2.2 Definition

The δ-interior [18] of a subset A of X is the union of all regular open sets of X contained in A and is denoted by $\text{int}_\delta(A)$. The subset A is called δ-open [18] if $A = \text{int}_\delta(A)$, i.e. a set is δ-open if it is the union of regular open sets, the complement of δ-open is called δ-closed. Alternatively, a set $A \subseteq X$ is δ-closed if $A = \text{cl}_\delta(A)$, where $\text{cl}_\delta(A) = \{x \in X : \text{int(cl(U))} \cap A \neq \phi,\ U \in \tau \text{ and } x \in U\}$.

2.3 Definition

A subset A of a topological space (X, τ) is called

1) generalized closed (briefly g-closed) [11] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).
2) $\hat{g}$-closed [17] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in (X, τ).
3) δ-generalized closed (briefly δg-closed) [6] if $\text{cl}_\delta(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).
4) $\delta\hat{g}$-closed [9] if $\text{cl}_\delta(A) \subseteq U$ whenever $A \subseteq U$ and U is $\hat{g}$-open in (X,τ).
5) δ-generalized star closed (briefly δg^*-closed) [15] if $\text{cl}_\delta(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open in (X,τ).
6) Weakly δ-generalized star closed (briefly $w\delta g^*$-closed) [14] if $\text{cl}_\delta(A) \subseteq U$ whenever $A \subseteq U$ and U is g*-open in (X,τ).
7) locally closed [7] if $A = U \cap F$, where U is open and F is closed in (X, τ).

2.4 Definition

A subset A of a topological space (X, τ) is called

1) $T_{3/4}$-space [11] if every δg-closed subset of (X,τ) is δ-closed in (X,τ).
2) ${}_{\delta g^*}T_\delta$-space [14] if every δg^*-closed subset of (X,τ) is δ-closed in (X,τ).
3) ${}_{\delta g}T_{\delta g^*}$-space [14] if every δg-closed subset of (X,τ) is δg^*-closed in (X,τ).
4) ${}_{\delta \hat{g}}T_{\delta g^*}$-space [14] if every $\delta\hat{g}$-closed subset of (X,τ) is δg^*-closed in (X,τ).
5) ${}_{w\delta g^*}T_\delta$-space [14] if every $w\delta g^*$-closed subset of (X,τ) is δ-closed in (X,τ).

6) ${}_{w\delta g^*}T_{\delta g^*}$-space [14] if every $w\delta g^*$-closed subset of (X,τ) is δg^*-closed in (X,τ).
7) ${}_{\delta g}T_{w\delta g^*}$-space [14] if every δg-closed subset of (X,τ) is $w\delta g^*$-closed in (X,τ).

2.5 Definition

A map $f : X \to Y$ is called

1) g-continuous [1] if $f^{-1}(V)$ is g-closed in (X, τ) for every closed set V of (Y, σ).
2) δg^*-continuous [16] if $f^{-1}(V)$ is δg^*-closed in (X, τ) for every closed set V of (Y, σ).
3) $w\delta g^*$-continuous [14] if $f^{-1}(V)$ is $w\delta g^*$-closed in (X, τ) for every closed set V of (Y, σ).
4) totally δg^*-continuous [14] if $f^{-1}(V)$ is δg^*-clopen in (X, τ) for every open subset V of (Y, σ).
5) totally $w\delta g^*$-continuous [14] if $f^{-1}(V)$ is $w\delta g^*$-clopen in (X, τ) for every open subset V of (Y, σ).
6) Strongly totally δg^*-continuous [14] if $f^{-1}(V)$ is δg^*-clopen in (X, τ) for every subset V of (Y, σ).
7) Strongly totally $w\delta g^*$-continuous [14] if $f^{-1}(V)$ is $w\delta g^*$-clopen in (X, τ) for every subset V of (Y, σ).
8) LC-continuous [7] if $f^{-1}(V)$ is a lc set of (X, τ) for every open set V of (Y, σ).

2.6 Definition

A map $f : X \to Y$ is called

1) Irresolute [4] if $f^{-1}(V)$ is a semi-open set of (X,τ) for every semi-open set V of (Y,σ).
2) δg^*- Irresolute [14] if $f^{-1}(V)$ is a δg^*-open set of (X,τ) for every δg^*-open set V of (Y,σ).
3) LC-irresolute [7] if $f^{-1}(V)$ is a lc-open set of (X,τ) for every lc-open set V of (Y,σ).

2.7 Definition [14]

A map $f : X \to Y$ is called
1) δg^*-closed map if the image of each closed set in X is δg^*-closed in Y.
2) $w\delta g^*$-closed map if the image of each closed set in X is $w\delta g^*$-closed in Y.

2.8 Definition [14]

A bijection map $f : X \to Y$ is called a

1) δg^*-homeomorphism if f is both δg^*-continuous and δg^*-closed map.
2) $\delta g^* c$-homeomorphism if f is both δg^*-irresolute and δg^*-irresolute.
3) $w\delta g^*$-homeomorphism if f is both $w\delta g^*$-continuous and $w\delta g^*$-closed map.

2.9 Definition [14]

Let A be a subset of X. Then A is called

1) $\delta g^* lc$-set if there exist a δg^*-open set U and a δg^*-closed set F of X such that $A = U \cap F$
2) $\delta g^* lc^*$-set if there exist a δg^*-open set U and a δ-closed set F of X such that $A = U \cap F$
3) $\delta g^* lc^{**}$-set if there exist a δ-open set U and a δg^*-closed set F of X such that $A = U \cap F$

The collection of all $\delta g^* lc$-sets (resp. $\delta g^* lc^*$-sets, $\delta g^* lc^{**}$-sets) of X will be denoted by $\delta G^*LC(X,\tau)$ (resp. $\delta G^*LC^*(X,\tau)$, $\delta G^*LC^{**}(X,\tau)$).

3. δg^*-Closed Sets in Digital Line

Digital topology is a term that has arisen for the study of geometric and topological properties of digital images. Digital topology was first studied by the computer image analysis researcher Azriel Rosenfeld [2]. Digital topology consists in providing algorithmic tools for Pattern Recognition, Image Analysis and Image processing using a discrete formulation for geometrical objects. It is applied in image processing. The study of images begins with Jordan Curve Theorem, an important one in the theory of computer graphics.

Khalimsky et al. [8] proved the Jordan Curve Theorem on the digital plane. The example of a digital plane is our computer screen. The basic building block of a digital plane is the digital line, a typical example of connected ordered topological spaces [COTS]. The digital line Z, equipped with the topology $\mathcal{K}$ generated by
$G_{\mathcal{K}} = \{\{2n-1, 2n, 2n+1\} : n \in Z\}$
The digital line is not T_1 however all non closed points are regular open. Dontchev et al. [6] introduced a new separation axiom $T_{3/4}$, as the class of topological spaces when every δg-closed sets is δ-closed that is closed in the semi-regularization in the topology.

It is showed by him the Khalimsky line is the $T_{3/4}$-space but not T_1. This section is devoted to the study of δg^*-closed sets of the digital line. The singleton sets in $(Z, \mathcal{K})$ is analyzed by many researchers, Devi et al.,[5], Maki [12], Arokyarani [1] Vigneswaran [19]. A parallel study is considered using δg^*-closed sets and it is proved here that (Z, $\mathcal{K}$) is a typical example of ${}_{\delta g*}T_\delta$-space, a ${}_{\delta g}T_{\delta g*}$-space and a ${}_{\delta \hat{g}}T_{\delta g*}$-space for 4 spaces. In (Z, $\mathcal{K}$) every δg^*-closed set is closed. Finally characterization of singleton points in the digital line (Z, $\mathcal{K}$) via the concepts of δg^*-closure, $w\delta g^*$-closure and δg^*-kernel are obtained.

First we recall related definitions and some properties of the digital line. The digital line or so called The Khalimsky line is the set of integers Z, equipped with the topology $\mathcal{K}$ generated by $\{\{2m-1, 2m, 2m+1\} : m \in Z\}$ as a subbase. The concept of the digital line is initiated by Khalimsky [8].

3.1 Lemma

Let (Z, K) be the digital line

1. If m is even, then cl(m) = {m}, int (m) = ϕ
2. If m is odd, then cl(m) = {m – 1, m, m+1} and int(m) = {m}.

The digital line contains both odd and even points. That is in general $Z = Z_E \cup Z_O$ where $Z_E = \{x / x \text{ is even in } Z\}$ and $Z_O = \{x / x \text{ is odd in } Z\}$

Similarly we can define for any subset A of (Z, $\mathcal{K}$), $A_E = \{x / x \text{ is even in } Z\}$ and $A_O = \{x / x \text{ is odd in } Z\}$

3.2 Definition

Let U(x) be the neighborhood of the point *x*

(a) If x is odd U(x) = {x}
(b) If x is even, then U(x) = {x – 1, x, x + 1}

3.3 Remark

In the digital line (Z, $\mathcal{K}$),

(a) If the corner points are odd then it is called open.
(b) If the corner points are even, then it is called closed.

3.4 Lemma

A subset A of (Z, $\mathcal{K}$) is open if and only if $2m \pm 1 \in A$ whenever $2m \in A$.

Proof: (Necessity) Let $2m \in A$. Since A is open, U(2m) = {2m-1, 2m, 2m+1} $\subseteq$ A.

(Sufficiency) To prove that A = int (A). Let $x \in A$.

Case 1. x = 2m. By the hypothesis $2m \pm 1 \in A$ and therefore U(2m) $\subseteq$ A. This implies x $\in$ int (A).

Case 2 x = 2m + 1. Since {2m+1} is an open subset of Z, x $\in$ int (A).

3.5 Lemma

A subset A of (Z, $\mathcal{K}$) is not closed if and only if there exists 2m + 1$\in$A such that 2m or $2m \pm 2 \notin A$..

Proof: (Necessity) A is not close implies A^c is not open. Therefore, by the above Lemma there exists $2m \in A.^c$ such that $2m-1$ or $2m+1 \notin A^c$.

Case 1 $2m+1 \notin A^c$ Then 2m + 1 $\in$ A and $2m \notin A.^c$.

Case 2 $2m-1 \notin A^c$ Then 2m – 1 $\in$ A and $2m \notin A$. Thus there exists 2m + 1 $\in$ A such that 2m or $2m+2 \notin A$..

(Sufficiency) Let there exist 2m + 1 $\in$ A such that 2m or $2m+2 \notin A$.. Then 2m or $2m+2 \in A^c$. and $2m+1 \notin A^c$. Therefore by Lemma 1, A^c is not open and hence A^c is not closed.

3.6 Lemma

GO(Z, $\mathcal{K}$) = $\mathcal{K}$

Proof: (Z, $\mathcal{K}$) is a $T_{1/2}$ space which is not T_1, because every singleton of (Z, $\mathcal{K}$) is open or closed. Therefore the class of g-open sets coincides with the open sets in (Z, $\mathcal{K}$). Hence GO(Z, $\mathcal{K}$) = $\mathcal{K}$

3.7 Lemma

Every singleton is either regular open or closed in (Z, $\mathcal{K}$).

Proof: Let {x} be not closed in (Z, $\mathcal{K}$). Then it is open in (Z, $\mathcal{K}$). Hence x = 2m + 1 for some integer m. Then int(cl{m}) = int (cl{2m+1}) = int $\{2m-1, 2m, 2m+1\}$ by Lemma 10.2.1 = {2m+1} = {x}. Therefore {x} is regular open.

3.8 Lemma

(Z, $\mathcal{K}$) is a $T_{3/4}$-space.

Proof: Dontchev et al [6] have proved in the Theorem 4.3 the following statements are equivalent.

1) X is a $T_{3/4}$-space
2) Every singleton {x} is regular open or closed

Hence the proof follows from Lemma 3.7.

3.9 Theorem

(Z, $\mathcal{K}$) is a ${}_{\delta g*}T_\delta$-space.

Proof: The digital line is proved to be a $T_{3/4}$-space by Dontchev et al [6] (Theorem 4.3 and Example 4.6). In $T_{3/4}$-space every δg-closed set is δ-closed. Let A be a δg^*-closed set in (Z, $\mathcal{K}$). Generally, in any space δg^*-closed set is δg-closed. Hence A is δg-closed. As (Z, $\mathcal{K}$) is a $T_{3/4}$-space, A is δ-closed. Therefore (Z, $\mathcal{K}$) is a ${}_{\delta g*}T_\delta$-space.

3.10 Theorem

(Z, $\mathcal{K}$) is a ${}_{\delta g}T_{\delta g*}$-space.

Proof: Consider (Z, $\mathcal{K}$), the digital line. It is a $T_{3/4}$-space every δg-closed set is δ-closed. Let A be a δg -closed set in (Z, $\mathcal{K}$). Since (Z, $\mathcal{K}$) is a $T_{3/4}$-space, A is δ-closed. In any space δ-closed set is δg^*-closed, A is δg^*-closed. Therefore (Z, $\mathcal{K}$) is a ${}_{\delta g}T_{\delta g^*}$-space.

3.11 Theorem

(Z, $\mathcal{K}$) is a ${}_{\delta\hat{g}}T_{\delta g^*}$-space.

Proof: Consider (Z, $\mathcal{K}$), the digital line. Let A be a $\delta\hat{g}$-closed set in (Z, $\mathcal{K}$). Generally, in any space $\delta\hat{g}$-closed set is δg-closed. Hence A is δg-closed. As (Z, $\mathcal{K}$) is a $T_{3/4}$-space, A is δ-closed. Therefore (Z, $\mathcal{K}$) is a ${}_{\delta\hat{g}}T_{\delta g^*}$-space.

3.12 Theorem

In the digital line (Z, $\mathcal{K}$) the following are equivalent

(a) $\mathcal{K}_\delta = \delta g^*\mathcal{K}^\#$ holds

(b) Every singleton $\{x\}$ is either g-closed or δ-open

(c) Every singleton $\{x\}$ is either g-closed or regular open

Proof: Proof follows from Theorem 3.3[15]

3.13 Corollary

Dontchev [6] says in lemma 4.2, every closed set in (Z, $\mathcal{K}$) is a δg^*-closed set.

3.14 Theorem

Let A be a subset of (Z, $\mathcal{K}$) and x be a point of (Z, $\mathcal{K}$). If A is a δg^*-closed set of (Z, $\mathcal{K}$) and $x \in A_0$, then $cl(\{x\}) - \{x\} \subseteq A$ and hence $cl(\{x\}) \subseteq A$ in (Z, $\mathcal{K}$).

Proof: Since $x \in A_0$, x = {2n + 1} for any n ∈ Z. Then $cl(\{x\}) = \{2n, 2n+1, 2n+2\}$. Therefore $cl(\{x\}) - \{x\} = \{2n, 2n+1, 2n+2\} - \{2n+1\} = \{2n, 2n+2\}$.

It is noted from the above theorem {2n} and {2n+2} are δg^*-closed points of (Z, $\mathcal{K}$). Suppose that $\{2n\} \not\subseteq A$ or $\{2n+2\} \not\subseteq A$. Let $2n+2 = x^+$ and $2n = x^-$ If $x^+ \not\in A$ then $x^+ \in cl\{x\} \subseteq cl(A)$ and so $x^+ \in cl(A) - A$. Since x^+ is an even number, $\{x^+\}$ is a non-empty g-closed set by Lemma 3.6. This is a contradiction to A is δg^*-closed [15]. Similarly, we can also prove the result for x^-. Therefore $x^+ \in A$ and $x^- \in A$ and hence $cl(\{x\}) - \{x\} \subseteq A$ and we have $cl(\{x\}) \subseteq A$ because $x \in A_0 \subseteq A$.

3.15 Theorem

Let B be a non-empty subset of (Z, $\mathcal{K}$). If $B_E = \phi$, then B is a δg^*-open set of (Z, $\mathcal{K}$).

Proof: Let F be a g-closed set of (Z, $\mathcal{K}$) such that $F \subseteq B$ since $B_E = \phi$ we have obviously that $B = B_0$ and so, $F \subseteq B_0$. It is obtained that F = ϕ because F is g-closed and therefore closed in (Z, $\mathcal{K}$) with $F \subseteq B$. Now F = ϕ ⊆ int(B). Therefore B is δg^*-open in (Z, $\mathcal{K}$).

3.16 Theorem

Let B be a non-empty subset of (Z, $\mathcal{K}$). For a subset B such that $B_E \neq \phi$, if a subset B is a δg^*-open set of (Z, $\mathcal{K}$) then $(U\{x\})_0 \subseteq B$ holds for each point $x \in B_E$.

Proof: Let $x \in B_E$. Since $\{x\}$ is closed, then $\{x\}$ is g-closed and $\{x\} \subseteq B$. Since B is δg^*-open, $\{x\} \subseteq int_\delta(A)$ by Theorem 4.4[15]. As $int_\delta\{x\} \subseteq int_\delta(int_\delta(B)) = int_\delta(B)$ and int{x} = U(x). We have U(x) ⊆ int(B). We can set x = 2n for any n ∈ Z as $x \in B_E$. Then U{x}= U(2n) = {2n-1, 2n, 2n+1}.

Then $(U\{x\})_0 = \{y \in U(x) / y \text{ is odd}\} = \{2n-1, 2n+1\}$.

$\{2n-1, 2n+1\} \subseteq B$ and so $\{2n-1, 2n+1\} \cap B \neq \phi$.

Therefore $(U\{x\})_0 \subseteq B$.

3.17 Theorem

Let A be a δg^*-closed set in (Z, $\mathcal{K}$). Then

(1) $(cl_\delta(A))_0 = \phi$ if $A = A_E$

(2) $(cl_\delta(A))_0 \neq \phi$ if $A = A_0 \cup A_E$

Proof: (1) Let $A = A_E = \{2n\}$ for any n ∈ Z and $\{2n\} \subseteq \{2n-1, 2n, 2n+1\} = U$, where U is g-open. By assumption $cl_\delta(\{2n\}) \subseteq U$. Then $cl_\delta(\{2n\})_0 = \phi$.. Thus $(cl_\delta(A))_0 = \phi$.

(2) Consider the smallest set A containing both odd and even integers such that $A = A_0 \cup A_E$

Case 1: $A \subseteq \{2n-1, 2n, 2n+1\}$ and take a g-closed U containing A. i.e., A ⊆ U. Since A is δg^*-closed in (Z, $\mathcal{K}$), $cl_\delta(A) \subseteq U$:

$cl_\delta(A) \subseteq \{2n-2, 2n-1, 2n, 2n+1, 2n+2\} \subseteq U$

$(cl_\delta(A))_0 \subseteq \{2n-1, 2n+2\} \neq \phi$

Case 2: $A \subseteq \{2n, 2n+1, 2n+2\}$ and take a g-closed set U containing A. i.e., A ⊆ U. Since A is δg^*-closed in (Z, $\mathcal{K}$), $cl_\delta(A) \subseteq U$: $cl_\delta(A) \subseteq \{2n, 2n+1, 2n+2\} \subseteq U$

$(cl_\delta(A))_0 \subseteq \{2n+1\} \neq \phi$.

3.18 Definition

For any subset A of (Z, $\mathcal{K}$), $\delta g^* - \ker(A) = \cap\{V / V \in \delta G^*O(Z, \mathcal{K}), A \subseteq V\}$

3.19 Theorem

For a topological space (Z, $\mathcal{K}$), we have the properties on the singleton as follows.

Let x be a point of Z and n ∈ Z.

1. If $x \in (Z)_0$, then $\delta g^* - \ker(\{x\}) = \{x\}$ and $\delta g^* - \ker(\{x\}) \in \delta G^*O(Z, \mathcal{K})$

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2. If $x \in (Z)_E$, then $\delta g^* - \ker(\{x\}) = \{x\} \cup (\cup\{x\})_0$ and $\delta g^* - \ker(\{x\}) \in \delta G^* O(Z, \mathcal{K})$

Proof:

1. For a point $x \in (Z)_0$, then by Theorem 3.15, $\{x\}$ is a δg^*-open set and from Definition 3.18, $\delta g^* - \ker(\{x\}) = \{x\}$ and $\delta g^* - \ker(\{x\}) \in \delta G^* O(Z, \mathcal{K})$

2. Let B be any δg^*-open set of (Z, $\mathcal{K}$) containing the point $x = \{2n\} \in (Z)_E$, Then by Theorem 3.16., $\{x\} \cup (\cup\{x\})_0 \subseteq B$ hold and $\{x\} \cup (\cup\{x\})_0 \in \delta G^* O(Z, \mathcal{K})$ Thus we have $\delta g^* - \ker(\{x\}) = \cap(\{x\}/\{x\} \subseteq V \in \delta G^* O(Z, \mathcal{K})$ = $\{x\} \cup (\cup\{x\})_0 = \{2n-1, 2n, 2n+1\}$. Then $\{x\} \cup (\cup\{x\})_0$ is open in (Z, $\mathcal{K}$). The kernel $\delta g^* - \ker(\{x\}) \in \delta G^* O(Z, \mathcal{K})$

3.20 Theorem

1. If $x \in (Z)_0$, then $\delta g^* - cl(\{x\}) = \{2n, 2n+1, 2n+2\}$, where x = {2n+1}
2. If $x \in (Z)_E$, then $\delta g^* - cl(\{x\}) = \{x\}$,
3. If $x \in (Z)_0$, then $\delta g^* - int(\{x\}) = \{x\}$,
4. If $x \in (Z)_E$, then $\delta g^* - int(\{x\}) = \phi$.

Proof: Follows from the above proved results.

3.21 Proposition [14]

- The digital line (Z, $\mathcal{K}$) is ${}_{w\delta g*}T_{\delta}$, ${}_{w\delta g*}T_{\delta g*}$ and ${}_{\delta g}T_{w\delta g*}$ - space
- In the digital line (Z, $\mathcal{K}$) the composition of δg^*-continuous functions is preserved. (By Proposition 4.2.17)
- In the digital line (Z, $\mathcal{K}$), every totally $w\delta g^*$ continuous function is totally δg^*-continuous. (By Proposition 4.7.5)
- In the digital line (Z, $\mathcal{K}$), every totally $w\delta g^*$ continuous function is strongly totally δg^*-continuous. (By Proposition 4.7.15)
- If a map f : (Z, $\mathcal{K}$) (Y, σ) $\xrightarrow{\text{from}}$ the digital line (Z, K) is δ-closed and surjective δg^*-irresolute then (Y, σ) is also a digital line. (By theorem 5.2.23)
- In the digital line (Z, $\mathcal{K}$), the composition of δg^*-closed maps is preserved. (By proposition 6.2.17)
- In the digital line (Z, $\mathcal{K}$), the composition of δg^*-homeomorphisms is preserved. (By proposition 6.3.22)
- Every δg^*-homeomorphism from the digital line to the digital line is a homeomorphism. (By Theorem 6.3.21)
- Every δg^*-homeomorphism from the digital line to the digital line is a $\delta g^* c$-homeomorphism. (By Theorem 6.3.23)
- In the digital line (Z, $\mathcal{K}$), the composition of $w\delta g^*$-closed maps is preserved. (By proposition 6.4.17)
- Every $w\delta g^*$-homeomorphism from the digital line to the digital line is a homeomorphism. (By Theorem 6.5.27)
- In the digital line (Z, $\mathcal{K}$), $\delta G^* LC(X, \tau)$ = $\delta LC(X, \tau)$ = $\delta G^* LC^*(X, \tau)$ = $\delta G^* LC^{**}(X, \tau)$ (By Proposition 7.2.9)
- In the digital line (Z, $\mathcal{K}$), δLC-continuity coincides with $\delta G^* LC$-continuity (resp. $\delta G^* LC^*$ continuity, $\delta G^* LC^{**}$-continuity) (By Proposition 7.3.7)
- In the digital line (Z, $\mathcal{K}$),
  (a) $\delta G^* LC$-continuity + contra δ continuity = $\delta G^* LC$-irresolute (By Proposition 7.3.12)
  (b) $\delta G^* LC^*$-continuity + contra δ continuity = $\delta G^* LC^*$-irresolute (By Proposition 7.3.14)
  (c) $\delta G^* LC$-continuity + contra δg^* irresolute = $\delta G^* LC$-irresolute (By Proposition 7.3.13)
- (Z, $\mathcal{K}$) is δg^* submaximal if and only if $\mathcal{P}(Z) = \delta G^* LC(Z, \mathcal{K})$ (By Proposition 7.2.26)
- (Z, $\mathcal{K}$) is δg^* submaximal every map in (Z,K) is $\delta G^* LC$-irresolute. (By Proposition 7.3.15)
- (Z, $\mathcal{K}$) is δg^* submaximal if and only if $\mathcal{P}(Z) = W\delta G^* LC(Z, \mathcal{K})$ (By Proposition 7.4.18)
- In the digital line (Z, $\mathcal{K}$), $W\delta G^* LC(X, \tau)$ = $\delta LC(X, \tau) = W\delta G^* LC^*(X, \tau) = W\delta G^* LC^{**}(X, \tau)$ (By Proposition 7.4.7)
- In the digital line (Z, $\mathcal{K}$) the following are equivalent
  (a) $A \in W\delta G^* LC(X, \tau)$
  (b) $A = U \cap w\delta g^* cl(A)$ for some $w\delta g^*$-open set U in (X, τ)
  (c) $w\delta g^* cl(A) - A$ is $w\delta g^*$-closed
  (d) $A \cup (X - w\delta g^* cl(A))$ is $w\delta g^*$ open (By Proposition 7.4.8)
- In the digital line (Z, $\mathcal{K}$),
  (a) $W\delta G^* LC$-continuity + contra δ continuity = $W\delta G^* LC^*$-irresolute (By Proposition 7.5.14)
  (b) $W\delta G^* LC^*$-continuity + contra δ continuity = $W\delta G^* LC^*$-irresolute (By Proposition 7.5.16)
  (c) $W\delta G^* LC$-continuity + contra $w\delta g^*$ irresolute = $W\delta G^* LC$-irresolute (By Proposition 7.5.15)
- In the digital line (Z, $\mathcal{K}$), δ-connectedness coincides with δg^*-connectedness (by Theorem 6.6.14).

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