Optimal Reliability Systems - Common Cause Failures

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Abstract: ***System reliability is a major challenge in system design. Unreliable systems are not only major source of user frustration, they are also expensive. Unfortunately, with the large component count in today's large-scale systems, failures are quickly becoming the norm rather than the exception. The reliability characteristic such as probability of survival, mean time to failure, frequency of failures and mean down time depend on the design and topological layout of the system. Common cause failure is an important phenomenon for a system with failure dependent parts. This paper deals with a single unit system that is operating in an environment exposed to the hazards of common cases failures, under some general assumptions the optional replacement policies were developed with the help of proposed measure of cost differences for old and new system.***

Keywords: common cause failures (CCF's), mean time to failures (MTTF's), optimal reliability systems, the reliability.

1. Introduction

Developing optimal reliability systems is an important research and development activity particularly useful in future studies in as much as systems that are safe, economically constituted and operating failure free are ensured. Earlier works in this direction relate to development of optimal systems under varied conditions and assumptions (vide, for example, [1] - [10]). Among these systems, those undergoing technological change and therefore are new entrants into the market particularly attract current research interests. However, these studies do not take into consideration the hazardous effect of common cause failures (CCF's) to which a system could possibly be exposed to the importance of the CCF's and their impact on system's reliability could hardly be over emphasized. In this chapter we improve upon the results in Venugopal et. al., [9] by incorporating the additional parameter, namely, the CCF's. Supporting numerical work as a comparison study to bring out a qualitative analysis is also presented.

2. Assumptions and the Model

We consider a single unit system functioning in an environmental set-up, which is exposed to CCF's. A new system now enters into the market, consequent upon research and development activities. A natural assumption is that the acquisition costs of the new system are higher than the old system but with smaller maintenance and repair costs. The modeling formulation is done under the following assumptions and adopting the notation as given below.

3. Assumptions

a) Steady state solutions are considered (tacitly implying that the time span is taken to be infinite).
b) Both old and new systems suffer decreased mean time to failures (MTTF's) as the failures increase.
c) The per unit efficiency (in terms of MTTF's) of the old and new systems remains the same.
d) The salvage values of the new system are considerably larger than the old system.
e) The salvage values for both systems decrease as the failures increase.

4. Notation

Co = AC. of old system
C_N = A.C. of new System
Co exp (-n/d) = salvage value of the old system after'n' failures (n =1, 2...), d > 0 and constant, characterizing the decay - scale parameter.
Co exp (m/n+ 1) = Maintenance and repair costs (M.R.C) of old system after 'n' failures, m > o and constant, characterizing the increasing nature of cost (with more failures).
CN exp (- r/n+ 1) = M.R.C. for the new system, where 0 < m < r, as per Modeling assumption.
$R_{n,c}$ (t) = Reliability of the system exposed to CCF's as well as random causes after (n+ 1) failures.
Mo (n) = MTTF of the old system after (n–1) failures, with the natural assumption Mo (∞) 0.
With these assumptions and adopting the notation, we develop in the following section optimal reliability systems in terms of deriving optimal replacement stage (n*) beyond which repairs are recommended to be stopped and the system is to be replaced.

5. Optimal n*

Under the above modeling setup, we have.
Mo (n) > Mo (n + 1), n = 1, 2 ... (1)

The expected cost per unit time, after 'n' failures for the old system and with nil failures for the new system are respectively given by,
Eo = (Co + Co $e^{-m/n+1}$ - C, $e^{-n/d}$)/Mo (n+ 1)
And EN = (C_N + C_N e^{-r}) / M_N (1) (3)

Following the idea in Venugopal et. al. [9], (4)
We propose the measure**, C (n),** defined as
C (n) = $\left|E_o(n)-E_N\right|$ (4)
The obvious motivation of proposing C (n) is to minimize C (n), with respect to 'n' and hence obtain the optimal n^*

which is the optimal replacement stage with the new system. The following basic theorem is proved to accomplish the purpose.

6. Theorem

The n* which minimizes **C (n),** in (4):

(i) Satisfies the pair of inequalities
S (n) > C_N/ C_o and S (n-1) < C_N/C_o, (5)
Where

$$S(n)=\left\{\frac{1-\exp\left(-(n+1)/d\right)+\exp\left(-m/n+2\right)}{M_o\,(n+2)}\right.$$

$$\left.+\frac{1-\exp\left(-n/d\right)+\exp\left(-m/n+1\right)}{M_o\,(n+1)}\right\}\frac{M_N(1)}{2\left(1+e^{-r}\right)} \quad (6)$$

(ii) n* < ∞ and (iii) n* is unique

Proof:
C (n) being discrete in 'n', n^* is obtained through the equalities,
C (n+1) > c(n) (7)
And c(n) < c(n + 1) (8)
From (4) and (7), we obtain (after simplifications)
[E_o^2 (n+ 1) - E_o^2 (n)] > 2 E_N [E_o (n+ 1) - E_o (n)]
This leads to
[E_o (n+1) + E_o (n)] > 2E_N (9)
Once we notice that **[E_o (n+ 1) - E_o (n)] > 0**
Rearranging terms is (9), we obtain **S (n) > C_N / C_o.**
Similarly (4) in (8), we obtain **S (n-4) < C_N/C_o** (10)
Thus (i) in the theorem is proved. We now show that **S (n)** is strictly
Increasing in 'n' and further **S (n) →∞,** as n → ∞.
For the purpose, be consider **{S (n + 1) - S (n)}.**

$$S(n+1)-S(n)=\frac{M_N(1)}{2\left(1+e^{-r}\right)}\left\{\frac{1-e^{-(n+2)/d}+e^{-m/n+3}}{M_o(n+3)}\right.$$

$$+\frac{1-e^{-(n+1)/d}+e^{-m/n+2}}{M_o(n+2)}-\frac{1-e^{-(n+1)/d}+e^{-m/n+2}}{M_o(n+2)}$$

$$\left.-\frac{1-e^{-(n+1)/d}+e^{-m/n+1}}{M_o(n+1)}\right\}$$

$$=\frac{M_N(1)}{2(1+e^{-r})}$$

$$\left\{\left[M_o\ (n+1)\ \left(1-e^{-(n+2)/d}\right)\right]-\left[M_o\ (n+3)\ \left(1-e^{n-d}\right)\right]+\right.$$

$$\left.\left[M_o(n+1)e^{-m/n+3}-M_o(n+3)e^{-m}/n+1\right]\right\}/M_o(n+1)M_o(n+3)>0$$

Thus S (n) is strictly increasing and → ∞ since **M_o (∞) = 0.**

Hence **C_N/C_O** being finite, S(n) crosses the value **C_N/C_o** just once at a finite crossing stage. Thus (ii) and (iii) are established. The proof is complete.

7. Numerical (Companion) Study

For purpose of illustrative numerical work, the probability failure laws for both random failures and CCF's are assumed to be negative exponential with respective parameters λ_n (after (n+ 1) failures) and λ_o for both the old and new systems.

Then, **$R_{n,c}$ (t)** $e^{-(\lambda_n+\lambda_o)t}$
So that **Mo (n) = $\int_o^\infty R_{n,c}$ (t) dt (11)**
= 1 / (λ_n + λ_o) (12)
And **M_N (1) = 1/ λ_o** (13)
Let us choose **A = (1.1) $^{n-1}$, d=3, m=2 and r = 4.**With these values we tabulate the S (n) values for n=0, 1, 2... 10 and for different λo values, in the following (corrected to second decimal place).

Table 1: n Vs S (n)

# of Repairs (n)	λ_o:0	0.1	0.2	0.4	0.6	0.8	1.0
0	0.43	0.43	0.43	0.42	0.42	0.42	0.42
1	0.98	0.97	0.96	0.94	0.93	0.92	0.91
2	1.46	1.43	1.4	1.37	1.34	1.32	1.3
3	1.89	1.84	1.8	1.7	1.69	1.65	1.62
4	2.3	2.23	2.17	2.07	2.01	1.94	1.9
5	2.71	2.61	2.53	2.39	2.29	2.22	2.16
6	3.13	3.02	2.89	2.71	2.58	2.48	2.4
7	3.56	3.39	3.26	3.04	2.88	2.75	2.65
8	4.01	3.81	3.64	3.48	3.18	3.02	2.9
9	4.5	4.26	4.05	3.73	3.49	3.31	3.16
10	5.02	4.73	4.49	4.11.	3.83	3.61	3.43

Now, using Theorem 1 and the related values, the optimal n* values for different **C_N/ C_o** ratio values and A.0 values are tabulated below.

Table 2: C_N/C_O and λ_oVs n^* values

C_N / C_o	λ_o:0	0.1	0.2	0.4	0.6	0.8	1.0
1.5	2	2	2	2	2	2	2
2.0	3	3	3	3	3	4	4
2.5	4	4	4	5	5	6	6
3.0	5	5	6	6	7	7	8

8. A Qualitative Analysis

From table 2 values we see that n^* values increase with increase in C_N/C_o acquisition cost ratio values as well as λ_o (CCF-ratio) values. The interpretation is that as new systems are costlier larger numbers of repairs are allowed before optimal replacements are recommended. Further CCF rates have same effects in certain ranges, for example, when C_N/C_o = 2.5, n^* is 4 for λ_o values in (0, 0.2) while it is 5 and 6 respectively for λ_o values in (0.4, 0.6) and (0.8, 1.0). That is, as λ_o values are larger, i.e., as CCF's become more intense, n^* is also larger, which is in keeping with natural physical experience.

9. Conclusion

In conclusion we note that when **λ_o = 0,** i.e., is the absence of CCF's; the results in Venugopal et al., [9] are recovered as a special case of our results. The redundancy allocation problem is formulated with the objective of maximizing system reliability in the presence of common cause failures. These types of failures can be described as events that lead to simultaneous failure of multiple components due to a common cause. When common cause failures are considered, component failure times are not independent. Since common cause failure events may vary from one system to another, three different interpretations of the reliability estimation problem are presented. Optimization models are presented solutions and support the position that consideration of common cause failures will lead to different and preferred "optimal" design strategies.

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Author Profile

Dr. Dhananjaya Reddy was awarded PhD from S.V. University; Tirupati on 1998.He did MSc & Mphil in the subjects of Mathematics and Statistics. His areas of Specializations are Operations Research, Stochastic Process and Abstract Algebra etc. He has 25 years of teaching and research experience.