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Bianchi Type-II Cosmological Model in Presence of Bulk Stress with Varying- in General Relativity

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Abstract: ***Bianchi Type-II space time is investigated in presence of bulk stress given by Landau and Lifshitz. To get a solution, a supplementary condition between metric potentials is used. The viscosity coefficient of bulk viscous fluid is assumed to be a simple power function of mass density where as the coefficient of shear viscosity is consider as proportional to the scale of expansion in the model. The cosmological term is found to be a decreasing function of time, which is supported by results found from recent type Ia supernovae observations. Also some physical and geometrical properties of the model are discussed.***

Keywords: Bianchi Type-II space time, viscous fluid, variable cosmological constant.

1. Introduction

The Einstein's field equations has two parameters, the Newtonian gravitational constant G and the cosmological constant $\Lambda(t)$. The Newtonian constant of gravitation G plays the role of a coupling constant between geometry of space and matter in Einstein's field equation. In an evolving universe, it is natural to take this constant as a function of time. In the modern cosmological theories, the dynamic cosmological term $\Lambda(t)$ remains a focal point of interest as it solves the cosmological constant problem in a natural way. There is significant observational evidence towards identifying Einstein's cosmological constant $\Lambda(t)$ or a component of material content of the universe that varies slowly with time and space and so acts like $\Lambda(t)$.
Recent cosmological observations by the High-z Supernova term and the Supernova cosmological project [1 – 7],suggest the existence of a positive cosmological constant $\Lambda(t)$ with magnitude $\Lambda\left(\frac{G_h}{c^3}\right) \approx 10^{-123}$. These observations on magnitude and red-shift of type Ia Supernova suggest that our universe may be accelerating with a large function of the cosmological density in the form of the cosmological Λ- term. Earlier researchers on this topic, are contained in Zeldovich [8], Weinberg [9], Dolgov [10 – 11], Bertolami [12], Ratra & Peebles [13], Carrol, Press and Turner [14]. Some of the recent discussions on the cosmological constant and consequence on cosmology with a time varying cosmological constant have been discuss by Dolgov [15 – 17], Tsagas and Maartens [18],Vishwakarma[19 – 24], and Pradhan et. al. [25 – 30]. This motivates us to study the cosmological models in which Λ varies with time Cosmological scenarios with a time varying Λ have been proposed by several researchers. A number of models with different decay laws for the variations of cosmological term were investigated during the last two decades .Chen and Wu[31], Pavon [32 – 33], Carvalho et al. [34], Lima and Maia [35], Lima and Trodden [36], Arbab and Abdel-Rahman [37], Cunha and Santos [38], Carneiro and Lima [39]. Weinberger [40], Heller and Klimek [41], Misner [42 – 43] Collins and Stewart [44] have studied the effect of viscosity on the evolution of cosmological models. Xing-Xiang Wang [45] discussed Kantowski-Sachs string cosmological model with bulk viscosity in general relativity. Also several aspects of viscous fluid cosmological model in early universe have been extensively investigated by many authors. Bali R. et. al. [46].Have studied Bianchi Type-III string cosmological models with time dependent bulk viscosity. Adhav et al. [47 – 48] have studied Bianchi Type-V string cosmological model with bulk viscous fluid and Kantowski-Sachs cosmological model in general relativity. Recently Verma et.al [49] investigated spatially homogeneous bulk viscous fluid models with time dependent gravitational constant and cosmological term. Accelerating Bianchi Type-I universe with time varying G and Λ-term in general relativity have been investigated by Pradhan et al.[50] .

Recently Mete et.al.[51-53] have studied various aspects of cosmological models in general theory of gravitation. Katore et.al.[54] have investigated Bianchi–I inflationary universe in presence of massless scalar field with flat potential in general relativity.
In this paper a new anisotropic L.R.S. Bianchi type-II stiff fluid cosmological model with variable Λ-term has been investigated by assuming supplementary conditions $A = B^n$ where A and B are metric potential. The outline of this paper is as follows: Basic equation of model are given in Section.2, the solutions of the field equations is given in

Section.3.In section 4 and 5, we have discuss the model.We conclude our result in Section 6.

2. The Metric and Field Equations

In an orthonormal frame, the metric for Bianchi Type-II space-time in the LRS case is given by

$$ds^2 = \eta_{ij}\sigma^i\sigma^j, \eta_{ij} = \mathrm{diag}(-1,1,1,1), \quad (1)$$

where the Cartan bases σ^i are given by

$$\sigma^0 = dt, \sigma^1 = B(t)\omega^1, \sigma^2 = A(t)\omega^2,$$
$$\sigma^3 = A(t)\omega^3 \quad (2)$$

where $A(t)$ and $B(t)$ are the time dependent metric functions. Assuming x,y,z) as local coordinates, the differential one forms ω^i are given by

$$\omega^1 = dy + xdz,\ \omega^2 = dz,\ \omega^3 = dx \quad (3)$$

The Energy momentum tensor for viscous fluid distribution in the presence of bulk stress given by Landau and Lifshitz [55] as

$$T_i^j = (\varepsilon + \rho)v_i v^j + pg_i^j$$
$$-\eta(v_{i;}^j + v_{;i}^j + v^j v^l v_{i;l} + v_i v^l v_{;l}^j$$
$$-(\xi - \tfrac{2}{3}\eta)\theta(g_i^j + v_i v^j)\ , \quad (4)$$

where p *is the* isotropic pressure, ε is the density, η *and* ξ are two coefficients viscosity, and v^i is the flow vector satisfying the relations

$$g_{ij}v^i v^j = -1 \quad (5)$$

We choose the coordinates to be commoving, so that

$$v^1 = 0 = v^2 = v^3, v^4 = 1 \ (6)$$

The Einstein's field equations with time-dependent Λ (in gravitational units $c=1$, $G=1$) read as

$$R_i^j - \tfrac{1}{2}R\delta_i^j + \Lambda g_i^j = -8\pi\, T_i^j\ ,(7)$$

For the metric (1) and energy-momentum tensor (4) in commoving system of co-ordinates, the field equation (7) yields as

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4B_4}{AB} + \frac{1}{4}\frac{B^2}{A^4} + \Lambda$$
$$= -8\pi\left[p - 2\eta\frac{A_4}{A} - \left(\xi - \frac{2}{3}\eta\right)\theta\right] \ (8)$$

$$2\frac{A_{44}}{A} + \left(\frac{A_4}{A}\right)^2 - \frac{3}{4}\frac{B^2}{A^4} + \Lambda$$
$$= -8\pi\left[p - 2\eta\frac{B_4}{B} - \left(\xi - \frac{2}{3}\eta\right)\theta\right] \ (9)$$

$$\left(\frac{A_4}{A}\right)^2 + 2\frac{A_4B_4}{AB} - \frac{1}{4}\frac{B^2}{A^4} + \Lambda = 8\pi\varepsilon\ ,(10)$$

where suffix 4 at the symbols A and B denotes ordinary differentiation with respect to t and θ is the shear expansion give by

$$\theta = v_{;i}^i \ (11)$$

3. Solution of the field Equations

Equations (8) – (10) are three independent equations in seven unknowns

$A, B, \varepsilon, p, \eta, \xi$ & Λ. For the complete determinacy of the system, we assume that

$$A = B^n \quad (12)$$

and the coefficient of shear velocity is proportional to the scale expansion, i.e.

$$\eta \propto \theta, (13)$$

where n is real number.

With this extra conditions, equations (8) and (9) leads to

$$-\frac{A_{44}}{A} - \left(\frac{A_4}{A}\right)^2 + \frac{B_{44}}{B} + \frac{A_4B_4}{AB} + \frac{B^2}{A^4}$$
$$= 16\pi\eta\left(\frac{A_4}{A} - \frac{B_4}{B}\right) \ (14)$$

Condition (13) leads to

$$\eta = l\left(2\frac{A_4}{A} + \frac{B_4}{B}\right)\ ,(15)$$

where l is proportionality constant.

Equation (14) together with (12) and (15) leads to

$$\frac{B_{44}}{B} + \alpha\frac{B_4^{\ 2}}{B^2} = \frac{B^{2-4n}}{n-1}\ ,(16)$$

$$\text{where } \alpha = 2n + 16\pi l(2n+1) \ (17)$$

which can be rewritten as

$$\frac{d}{dB}\left(B^{2\alpha}B_4^{\ 2}\right) = \frac{2}{n-1}B^{2\alpha-4n+3}, (18)$$

which on integration gives

$$B_4^{\ 2} = \frac{1}{(n-1)(\alpha-2n+2)}B^{4(1-n)} + \frac{\beta}{B^{2\alpha}} \ (19)$$

where β is a constant of integration . After a suitable transformation of coordinates, the metric (1) reduces to the for

$$ds^2 = -\left[\frac{T^{4(1-n)}}{(n-1)(\alpha-2n+2)} + \frac{\beta}{T^{2\alpha}}\right]^{-1} dT^2 +$$
$$T^{2n}[dX^2 + dZ^2] + T^2[dY - Xdz]^2, (20)$$

The pressure and density for model (20) are given by

$$8\pi p = K_1 T^{2(1-2n)} + \frac{K_2}{T^{2(\alpha+1)}} +$$
$$+8\pi\xi(2n+1)\sqrt{\left[\frac{T^{2(1-2n)}}{(n-1)(\alpha-2n+2)} + \frac{\beta}{T^{2(\alpha+1)}}\right]} - \Lambda \ (21)$$

and

$$8\pi\varepsilon = K_3 T^{2(1-2n)} + \frac{\beta n(n+2)}{T^{2(\alpha+1)}} + \Lambda \ (22)$$

where,

$$K_1 = \frac{[(-n^2)(32\pi l-9) - n(16\pi l+3\alpha+12) - (16\pi l-3\alpha)]}{3(n-1)(\alpha-2n+2)}$$

$$K_2 = \frac{-\beta}{3}[n^2(3-32\pi l) + n(16\pi l - 3\alpha) + (16\pi l - 3\alpha)]$$

and $K_3 = \frac{6n^2 + n(4-\alpha) + (\alpha+2)}{4(n-1)(\alpha-2n+2)}$

For the specification of ξ ,now we assume that the fluid obeys an equation of state of

the form $p = \gamma\varepsilon$, (23)

where $\gamma(0 \leq \gamma \leq 1)$ is constant.

Thus, given $\xi(t)$ we can solve the cosmological parameters. In most of the investigation involving bulk viscosity is assume to be a simple power function of the energy density (Pavon, [32]; Maartens, [56]; Zimdahl, [57])

$$\xi(t) = \xi_0\rho^m, \quad (24)$$

where ξ_0 and m are constant. If $m = 1$ Equation (24) may correspond to a relative fluid (Weinberg[9]). However, more realistic models (Santos, [58]) are based on m lying in the regime $0 \leq m \leq \frac{1}{2}$.

Using (24) in (21), we obtain

$$8\pi p = K_1T^{2(1-2n)} + \frac{K_2}{T^{2(\alpha+1)}} + 8\pi\xi_0\varepsilon^m(2n+1)\sqrt{\frac{T^{2(1-2n)}}{(n-1)(\alpha-2n+2)} + \frac{\beta}{T^{2(\alpha+1)}}} - \Lambda \quad (25)$$

We consider the two model corresponding to *m=0* and *m=1*

3.1. Model- I :

When $m = 0$, Equation (24) reduces to $\xi(t) = \xi_0 =$ constant. Hence in this case Equation (25), with the use of (22) and (23), leads to

$$8\pi\varepsilon(1+\gamma) = (K_1+K_3)T^{2(1-2n)} + \frac{[K_2+\beta n(n+2)]}{T^{2(\alpha+1)}} + 8\pi\xi_0(2n+1)\sqrt{\frac{T^{2(1-2n)}}{(n-1)(\alpha-2n+2)} + \frac{\beta}{T^{2(\alpha+1)}}} \quad (26)$$

Eliminating $\varepsilon(t)$ between (26) and (22), we have

$$(1+\gamma)\Lambda = (K_1-\gamma K_3)T^{2(1-2n)} + \frac{[K_2-\gamma\beta n(n+2)]}{T^{2(\alpha+1)}} + 8\pi\xi_0(2n+1)\sqrt{\frac{T^{2(1-2n)}}{(n-1)(\alpha-2n+2)} + \frac{\beta}{T^{2(\alpha+1)}}} \quad (27)$$

3.2. Model-II

When $m = 1$, Equation (24) reduces to $\xi(t) = \xi_0\varepsilon$. Hence in this case Equation (25), with the use of (22) and (23), leads to

$$\varepsilon = \frac{1}{8\pi\left[1+\gamma-\xi_0(2n+1)\sqrt{\frac{T^{2(1-2n)}}{(n-1)(\alpha-2n+2)} + \frac{\beta}{T^{2(\alpha+1)}}}\right]} \times \left[(K_1+K_3)T^{2(1-2n)} + \frac{[K_2+\beta n(n+2)]}{T^{2(\alpha+1)}}\right] \quad (28)$$

Eliminating $\varepsilon(t)$ between Equation (28) and (22) , we have

$$\Lambda = \frac{1}{\left[1+\gamma-\xi_0(2n+1)\sqrt{\frac{T^{2(1-2n)}}{(n-1)(\alpha-2n+2)} + \frac{\beta}{T^{2(\alpha+1)}}}\right]} \times \left[(K_1+K_3)T^{2(1-2n)} + \frac{[K_2+\beta n(n+2)]}{T^{2(\alpha+1)}}\right] - \left[K_3T^{2(1-2n)} + \frac{\beta n(n+2)}{T^{2(\alpha+1)}}\right] \quad (29)$$

equation (27) and (29) , we observe that If $\alpha > 0, n > 0$ positive cosmological constant is a decreasing function of time and approaches a small value in the present epoch.

4. Some Physical Aspects of the Model

The straight forward calculation leads to the following expression for the scalar of expansion θ for the shear σ of the fluid for the metric (20)

$$\theta = (2n+1)\sqrt{\frac{T^{2(1-2n)}}{(n-1)(\alpha-2n+2)} + \frac{\beta}{T^{2(\alpha+1)}}} \quad (30)$$

$$\sigma = (2n+1)\sqrt{\frac{7}{18}\left(\frac{T^{2(1-2n)}}{(n-1)(\alpha-2n+2)} + \frac{\beta}{T^{2(\alpha+1)}}\right)} \quad (31)$$

The expansion factor θ decreases as a function of T and asymptotically approaches zero with ε and p also approaches zero as $T \to \infty$

Particular Model

If we set $n = 2$ the geometric space time (20) reduces to the form

$$ds^2 = -\left[\frac{1}{2(1+40\pi l)T^4} + \frac{\beta}{T^{8(1+20\pi l)}}\right]^{-1} dT^2 + T^4[dX^2 + dZ^2] + T^2[dY - Xdz]^2, \quad (32)$$

The pressure and density for model (32) are given by

$$8\pi p = -\frac{208\pi l}{3(1+40\pi l)T^6} + \frac{16\beta}{3}\frac{(34\pi l+3)}{T^{2(5+80\pi l)}} + 40\pi\xi\sqrt{\left[\frac{1}{2(1+40\pi l)T^6} + \frac{\beta}{T^{2(5+80\pi l)}}\right]} - \Lambda \quad (33)$$

$$8\pi\varepsilon = \frac{(13-40\pi l)}{4(1+40\pi l)T^6} + \frac{8\beta}{T^{2(5+80\pi l)}} + \Lambda \quad (34)$$

4.1. Model- I :

When $m = 0$, Equation (24) reduces to $\xi(t) = \xi_0 =$ constant. Hence in this case Equation (33), with the use of (23) and (34), leads to

$$8\pi\varepsilon(1+\gamma) = \frac{(39-952\pi l)}{12(1+40\pi l)T^6} + \frac{8\beta(60\pi l+9)}{3T^{2(5+80\pi l)}} + 40\pi\xi_0\sqrt{\left[\frac{1}{2(1+40\pi l)T^6} + \frac{\beta}{T^{2(5+80\pi l)}}\right]} \quad (35)$$

Eliminating $\varepsilon(t)$ between Equation (34) and (35) , we have

$$(1+\gamma)\Lambda = \frac{2(39-536\pi l)+(39-120\pi l)\gamma}{12(1+40\pi l)T^6} + \frac{8\beta(68\pi l-3\gamma+6)}{3T^{2(5+80\pi l)}} + 40\pi\xi_0\sqrt{\left[\frac{1}{2(1+40\pi l)T^6}+\frac{\beta}{T^{2(5+80\pi l)}}\right]} \quad (36)$$

4.2. Model-II :

When $m = 1$, Equation (24) reduces to $\xi(t) = \xi_0\varepsilon$. Hence in this case Equation (33), with the use of (23) and (34), leads to

$$\varepsilon = \frac{1}{8\pi\left[1+\gamma-5\pi\xi_0\sqrt{\left[\frac{1}{2(1+40\pi l)T^6}+\frac{\beta}{T^{2(5+80\pi l)}}\right]}\right]} \times \left[\frac{(39-952\pi l)}{12(1+40\pi l)T^6}+\frac{8\beta(68\pi l+9)}{3T^{2(5+80\pi l)}}\right] \quad (37)$$

Eliminating ε between equation (37)and (34)

$$\Lambda = \frac{1}{\left[1+\gamma-5\pi\xi_0\sqrt{\left[\frac{1}{2(1+40\pi l)T^6}+\frac{\beta}{T^{2(5+80\pi l)}}\right]}\right]} \times \left[\frac{(39-952\pi l)}{12(1+40\pi l)T^6}+\frac{8\beta(68\pi l+9)}{3T^{2(5+80\pi l)}}\right] - \left[\frac{(13-40\pi l)}{4(1+40\pi l)T^6}+\frac{8\beta}{T^{2(5+80\pi l)}}\right] \quad (38)$$

Some Physical aspect of the model

The scalar of expansion θ and the shear σ of the model (32) is given by

$$\theta = 5\sqrt{\left[\frac{1}{2(1+40\pi l)T^6}+\frac{\beta}{T^{10(1+16\pi l)}}\right]} \quad (39)$$

$$\sigma = 5\sqrt{\left[\frac{7}{36(1+40\pi l)T^6}+\frac{7\beta}{18T^{10(1+16\pi l)}}\right]} \quad (40)$$

5. Special Model

If we set $n = 2$ and $l = -\frac{1}{32\pi}$ equation (19) leads to

$$\frac{B^4}{(\beta B-2)}dB^2 = dt^2 \quad (41)$$

Using the transformation the metric (1) takes the form

$$ds^2 = -\frac{T^4}{(\beta T-2)}dT^2 + T^4[dX^2+dZ^2] + T^2[dY-Xdz]^2, \quad (42)$$

The pressure and density for the model (42) is given by

$$8\pi p = -\left(\frac{26}{T}+\beta\right)\frac{1}{3T^5} + 40\pi\xi\sqrt{\left(\beta-\frac{2}{T}\right)\frac{1}{T^5}} - \Lambda \quad (43)$$

$$8\pi\varepsilon = \left(8\beta-\frac{65}{4T}\right)\frac{1}{T^5} + \Lambda \quad (44)$$

5.1. Model- I :

When $m = 0$, Equation (24) reduces to $\xi(t) = \xi_0$= constant. Hence in this case Equation (43), with the use of (23) and (44), leads to

$$8\pi\varepsilon(1+\gamma) = \left(\beta-\frac{13}{4T}\right)\frac{23}{3T^5} + 40\pi\xi_0\sqrt{\left(\beta-\frac{2}{T}\right)\frac{1}{T^5}} \quad (45)$$

Eliminating $\varepsilon(t)$ between Equation (44) and (45) , we have

$$(1+\gamma)\Lambda = -\left(\frac{26}{T}+\beta\right)\frac{1}{3T^5} - \left(8\beta-\frac{65}{4T}\right)\frac{\gamma}{T^5} + 40\pi\xi_0\sqrt{\left(\beta-\frac{2}{T}\right)\frac{1}{T^5}} \quad (46)$$

5.2. Model-II

When $m = 1$, Equation (24) reduces to $\xi(t) = \xi_0\varepsilon$. Hence in this case Equation (43), with the use of (23) and (44), leads to

$$\varepsilon = \frac{1}{8\pi\left[1+\gamma-5\xi_0\sqrt{\left(\beta-\frac{2}{T}\right)\frac{1}{T^5}}\right]} \times \left(\beta-\frac{13}{4T}\right)\frac{23}{3T^5} \quad (47)$$

$$\Lambda = \frac{1}{\left[1+\gamma-5\xi_0\sqrt{\left(\beta-\frac{2}{T}\right)\frac{1}{T^5}}\right]} \times \left(\beta-\frac{13}{4T}\right)\frac{23}{3T^5} - \left(8\beta-\frac{65}{4T}\right)\frac{1}{T^5} \quad (48)$$

From the equation (46) and (48) we observe that the positive cosmological constant is a decreasing function of time and approaches small value.

Some Physical aspect of the model

The scalar of expansion θ and the shear σ of the model (42) is given by

$$\theta = 5\sqrt{\left[\frac{\beta T-2}{T^6}\right]} \quad (49)$$

$$\sigma = 5\sqrt{\frac{7}{18}\left[\frac{\beta T-2}{T^6}\right]} \quad (50)$$

6. Conclusion

We have presented Bianchi type-II non static cosmological model in presence of bulk stress given by Landau L.D. and Lifshitz E.M. It is found that physically relevant solutions are possible for the Bianchi-II space time with bulk stress in the presence of time varying Λ –term. For solving the field equations we have assumed that the fluid obey an equation of state of the form $p = \gamma\varepsilon$, and bulk viscous fluid is assumed to be the simple power function of mass density given by $\xi(t) = \xi_0\rho^m$. Generally the models are expanding, shearing and non-rotating. The cosmological constant in all models given in section 3.1 and 3.2 are decreasing function of time and all approaches a small positive value at finite large times (i.e., the present epoch). These results are supported by the results from the supernova observations recently obtain by the High-z Supernova team and Supernova Cosmological project [1-7].Thus with our approach, we obtain a physically relevant decay law for the cosmological constant unlike other investigators where adhoc laws were used to arrive at a mathematical expressions for the decaying energy. Thus our models are more general than those studied earlier. In all models, the physical parameters pressure and density are found to be decreasing function of time. Also we find that all the physical quantities the expansion scalar and the shear scalar**.**

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