# Performance Analysis of Multi-Carrier Modulation Techniques Using FFT, DWT and DT-WPT

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**Abstract***: **Multi-Carrier Modulation (MCM) is a very efficient transmission technology for very high data rate and for reliable wireless communication systems. Orthogonal Frequency Division Multiplexing (OFDM) is the proven technology comes under multi-carrier modulation. In this scheme number of orthogonal carriers are overlapped and transmitted over the channel. Traditional OFDM systems use Fast Fourier Transform (FFT) for orthogonal bases. Recent studies show wavelets can be used instead of FFT and systems perform better due to inherent properties of the wavelets. Dual Tree Wavelet Packet Transform (DT-WPT) is an enhancement to Discrete Wavelet Transform (DWT) with the additional properties of shift invariance and perfect reconstruction. In this paper we implemented OFDM schemes using FFT, DWT and DT-WPT and their performance is compared over AWGN channel and modulation employed is BPSK. BER vs SNR performance measure is plotted and compared for all these schemes using MATLAB tool.***

**Keywords:** Multi-Carrier Modulation (MCM), Orthogonal Frequency Division Multiplexing (OFDM), Fast Fourier Transform (FFT), Discrete Wavelet Transform (DWT), Dual Tree Wavelet Packet Transform (DT-WPT), Additive White Gaussian Noise (AWGN), BPSK, MATLAB

## 1. Introduction

Multi-Carrier Modulation (MCM) based on FFT has been adapted in many wireless communication systems. These include digital audio and terrestrial video broadcast (DAB, DVB-T), local area networks such as IEEE 802.11 a/g/n [1-3]. This is known as OFDM. OFDM is a technique of transmitting data by dividing the input into parallel sub streams and each modulated and finally multiplexed on to a channel at different carrier frequencies. At the very data rates as duration of each sub-carrier is larger than the original input, multi-path interference can be avoided. Orthogonality of sub-carriers is the main concept in OFDM. The property of orthogonality allows simultaneous transmission of additional sub-carriers in a tight frequency space without interference from each other. Wavelet Transform is capable of providing the time and frequency information simultaneously. Wavelets are known to have compact support (localization) both in time and frequency domain, and possesses better orthogonality. Wavelets can better combat narrowband interference and is more robust to inter-carrier interference (ICI) than FFT realization [2]. One more advantage of using wavelets is there is no need of cyclic-prefix (CP) addition, so that higher data rates can be possible than FFT realization.

Wavelet Packet Modulation (WPM) is a carrier-less [2] that uses a filtering and de-filtering technique to convey orthogonal multi sub-band information from transmitter to receiver. WPM shares all the benefits of multi-carrier technique and exhibits further benefit such as higher efficiency due to elimination of guard interval (GI) i.e. no cyclic-prefix (CP)[3] is needed. Unlike OFDM, which divides the whole bandwidth into orthogonal and overlapping sub-bands of equal bandwidths, WPM using the discrete wavelet packet transform is a multiplexing transmission method in which data is assigned to wavelet sub-bands having different time and frequency resolutions. Due to the use of wavelets the transmission power can be reduced [3-5, 14-15].

The WPM discussed above uses only real arithmetic, as opposed to the complex valued Discrete Fourier Transform (DFT). This reduces the signal processing complexity and power consumption, but it suffers from three major limitations: shift variance, poor directionality and absence of phase information. To solve these problems, Dual Tree Wavelet Packet Transform (DT-WPT) [4-6] can be used this uses complex arithmetic and overcomes the above stated drawbacks. In this paper DT-WPT is used in OFDM scheme and its performance in AWGN [4, 11-14] is compared with the OFDM schemes that are realized with FFT and DWT.

## 2. Wavelet Transforms

### 2.1 Discrete Wavelet Transform (DWT)

The original signal **s** is passed through low pass filter h(n) and high pass filter g(n). These filters are designed such that they should satisfy perfect reconstruction condition [7]. The outputs of these filters are down-sampled by a factor of 2. The outputs of low pass filter are called approximate coefficients Ca and that of high pass filter are detail coefficients Cd. DWT filter bank implementation is shown in Fig.1. In the next level of decomposition the approximate coefficients Ca obtained in the above step are passed through the same low pass and high pass filters and down-sampled to obtain next level approximate and detail coefficients and so on. After filtering, down-sampling is being used to avoid the redundancy [7]. These obtained coefficients form an orthogonal bases and the original signal can be reconstructed by applying Inverse Discrete Wavelet Transform (IDWT).

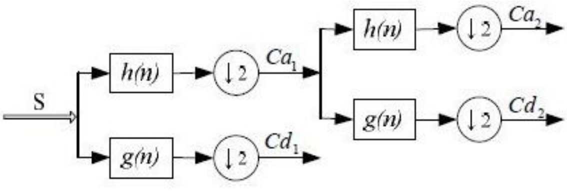

**Figure 1:** FB implementation of DWT

### 2.2 Wavelet Packet Transform (WPT)

In DWT in further level of decomposition only approximate coefficients are considered, where in WPT [7-11] both approximate and detail coefficients are passed through the low pass and high pass filters to get the next level coefficients, results in rich resolution. WPT filter bank implementation is shown in Fig.2. These obtained coefficients form an orthogonal bases and the original signal can be reconstructed by applying Inverse Wavelet Packet Transform (IWPT).

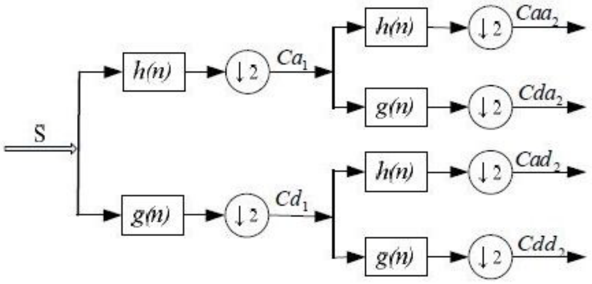

**Figure 2:** FB implementation of WPT

### 2.3 Stationary Wavelet Transform (SWT)

Both DWT and WPT suffers from the lack of shift invariance, i.e. small shift in input signal can cause major variations in energy distribution between the filter coefficients and may cause some error in reconstruction [7]. In SWT, this problem is carried out by removing down-samplers after filtering. As the down-samplers are removed, the same filters can't be used in the next level decomposition, these filters are up-sampled by inserting zeros between the filter coefficients [7]. SWT filter bank implementation is shown in Fig.3.

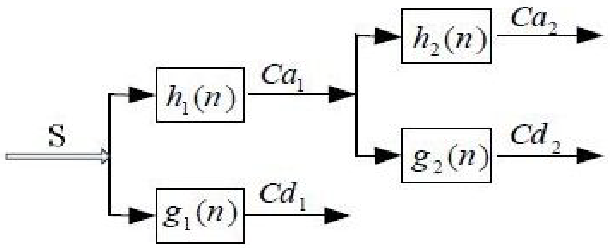

**Figure 3:** FB implementation of SWT

### 2.4 Dual Tree Wavelet Packet Transform (DT-WPT)

SWT solves the problem of shift invariance but it increases the redundancy by a factor of N*L, where N is the length of the input signal and L is the maximum level of decomposition. DT-WPT provides the solution to this without having high redundancy; it takes the advantage of complex filter values instead of real ones. DT-WPT has the properties of shift invariance, perfect reconstruction, limited redundancy [8-10]. DT-WPT uses two different sets of WPT filter banks, one for real values and other for imaginary values. DT-WPT filter bank implementation is shown in Fig.4.

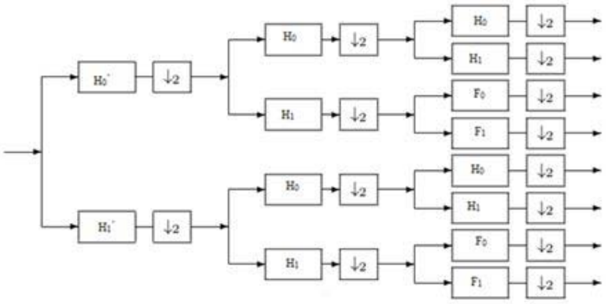

**Figure 4:** FB implementation of DT-WPT

The relation between $H^k(e^{jw})$ and $H'^k(e^{jw})$ is given as $H^k(e^{jw}) = \boldsymbol{H}\,(H'^k(e^{jw}))$, where ***H*** is Hilbert Transform [7].

The filter responses should satisfy Hilbert Transform pairs, means they should satisfy the following condition given in Eq.1, from this we need to set

$$f_i(n) = f_i'(n)$$

$$H^k(e^{jw})F_i\left(e^{j2^k w}\right) = \boldsymbol{H}(H'^k(e^{jw})F_i'\left(e^{j2^k w}\right)) \tag{1}$$

Using the above property the equivalent structure for kth sub-band decomposition of the first filter bank (FB) is shown in Fig.5 and kth stage of second filter bank (FB) is shown in Fig.6. The low pass and high pass filters of the first stage should satisfy $h_0'^{(1)}(n) = h_0^{(1)}(n-1)$ and $h_1'^{(1)}(n) = h_1^{(1)}(n-1)$. The second wavelet FB is obtained by replacing the first stage filters $h_i^{(1)}(n)$ by $h_i^{(1)}(n-1)$ and by replacing $h_i^{(1)}$ by $h_i'^{(1)}(n)$ for i ε {0,1}.

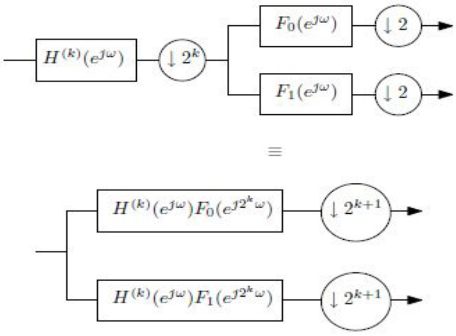

**Figure 5:** Equivalent structure of kth sub-band decomposition in first FB

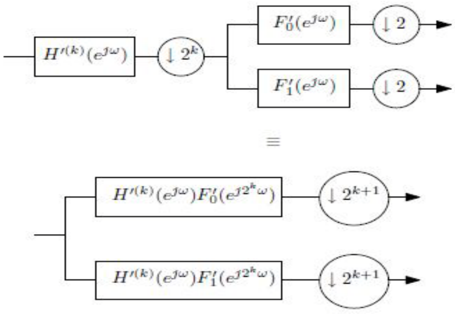

**Figure 6:** Equivalent structure of kth sub-band decomposition in second FB

## 3. Types of OFDM Systems

### 3.1 FFT based OFDM System

Fig.7 shows the block diagram of the basic OFDM system using FFT, Discrete Fourier Transform is efficiently implemented by Fast Fourier Transform (FFT). The input data is first modulated using the BPSK modulation block, and it is converted to parallel data. These symbols are now sent through IFFT block which generates N parallel data streams. Its output in time domain is given in Eq.2.

$$X_{k(n)} = \frac{1}{\sqrt{2}} \sum_{i=0}^{N-1} X_m(i) e^{j2\pi ni/N} \qquad (2)$$

To mitigate ISI, cyclix-prefix (CP) is added before the transission and this data is passed through AWGN channel. At the receiver , first cyclix-prefix (CP) is removed and is passed through FFT block, its output in frequency domain is demodulated to recover the transmitted data.

$$U_{m(i)} = \frac{1}{\sqrt{2}} \sum_{i=0}^{N-1} U_{k(n)}(i) e^{-j2\pi ni/N} \qquad (3)$$

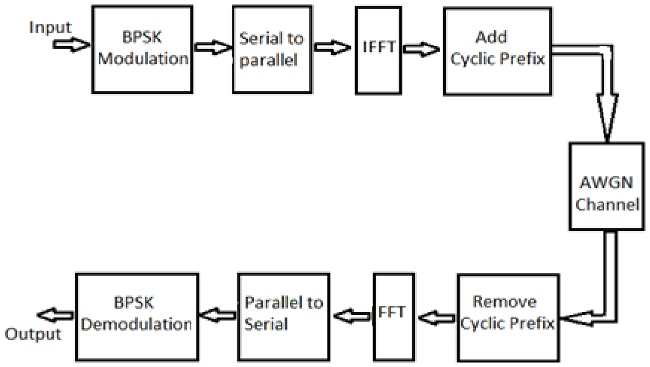

**Figure 7:** FFT based OFDM System

### 3.2 DWT based OFDM System

This is the alternate way of implementing the OFDM scheme using DWT as shown in Fig.8. DWT-OFDM replaces the time-windowed complex exponents that are employed in FFT-OFDM by wavelt carriers at different scales (j) and positions on the time axis (k).These functions are generated by translation and dilation of mother wavelet denoted by *ψ(t)*.

$$\psi_{j,n}(t) = 2^{-\frac{j}{2}}\psi(2^{-j}t - n) \qquad (4)$$

Wavelet carriers exhibit better time-frequency localization than complex exponentials. These functions have orthonormal basis of $L_2(R)$ if infinite number of scales are considered. To obtain finite number of scales, scaling function ϕ(t) is used. DWT-OFDM symbols can be considered as weighted sum of wavelet and scaled carriers, as expresses in Eq.5.

$$s(t) = \sum_{j=0}^{\infty}\sum_{-\infty}^{\infty} d(j,n).2^{j}.\psi(2^{j}t - n) + \sum_{n=-\infty}^{\infty} c(n).\phi(t-n) \qquad (5)$$

The scaling coefficient c(n) and wavelet coefficient d(j,n) are computed via the inner products given in the following equations.

$$c(t) = \int_{-\infty}^{\infty} s(t)\phi(t-n)dt \qquad (6)$$

$$d(j,n) = 2^{j/2}\int_{-\infty}^{\infty} s(t)\,\psi(2^{j}t - n)dt \qquad (7)$$

Due to higher spectral containment between the sub-channels, wavelet based OFDM is better able to overcome the effects of narrowband interference and is more robust to ICI than FFT filters. Wavelet OFDM can be implemented by overlapping waveforms to preserve the data rate Thus cyclic-prefix (CP) is not required in this scheme.

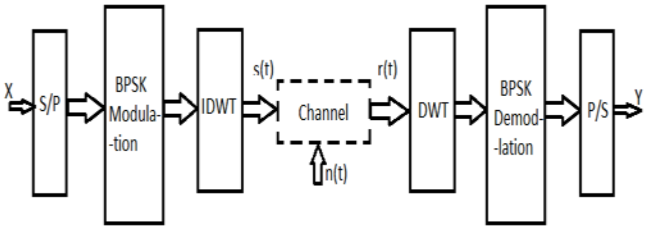

**Figure 8:** DWT based OFDM System

### 3.3 DT-WPT based OFDM System

DT-WPT is approximately analytic and is also approximately shift invariant, means the energy in each sub-band is preserved although the input sequence is shifted by an arbitrary number of samples. The approximate shift invariance property in DT-WPT is due to the reduction of aliasing compared to the above stated schemes. The aliasing caused by down-samplers is reduced in DT-WPT because of band-pass response of each branch is approximately analytic. Block diagram of DT-WPT based OFDM is shown in fig.9, here after serial to parallel conversion of the input data, it is BPSK modulated using the modulator block, it is then passed through Inverse Dual Tree Wavelet Packet Transform (IDT-WPT) block, and the resultant signal is then passed through the AWGN channel. The reverse operations are done at the receiver side, noise affected signal is passed through DT-WPT block, and then the data is demodulated to recover the transmitted data.

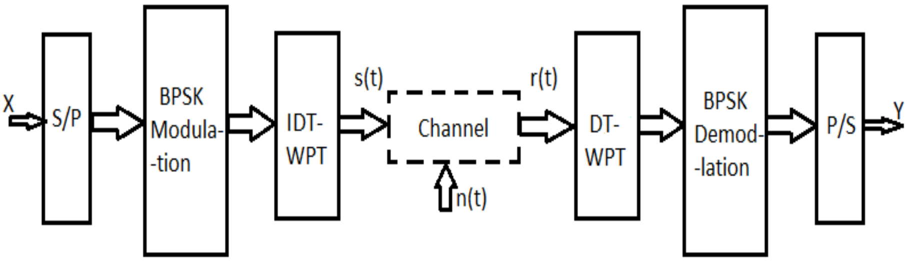

**Figure 9:** DT-WPT based OFDM System

## 4. Performance Analysis of Implemented Schemes

BER performance is simulated for OFDM scheme using FFT for different SNRs over AWG channel, the obtained results are plotted in Fig.10, BER vs SNR simulated for OFDM scheme using DWT with Haar wavelet is plotted in Fig.11 and that of OFDM scheme using DT-WPT is plotted in Fig.12.

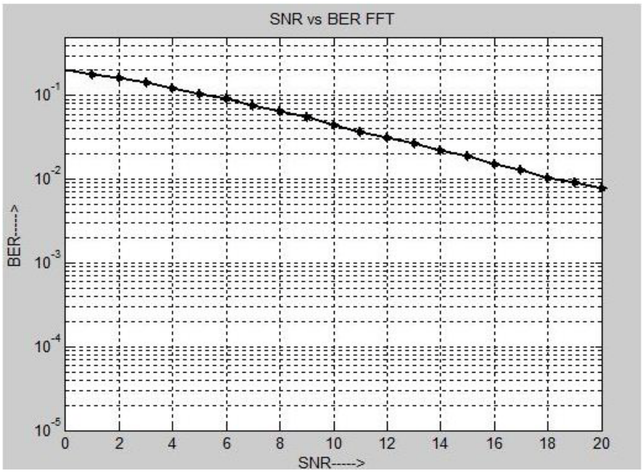

**Figure 10:** SNR vs BER for FFT based OFDM System

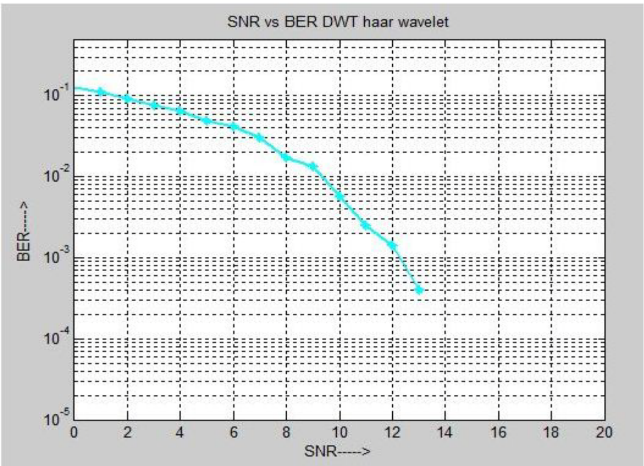

**Figure 11:** SNR vs BER for DWT (Haar) based OFDM System

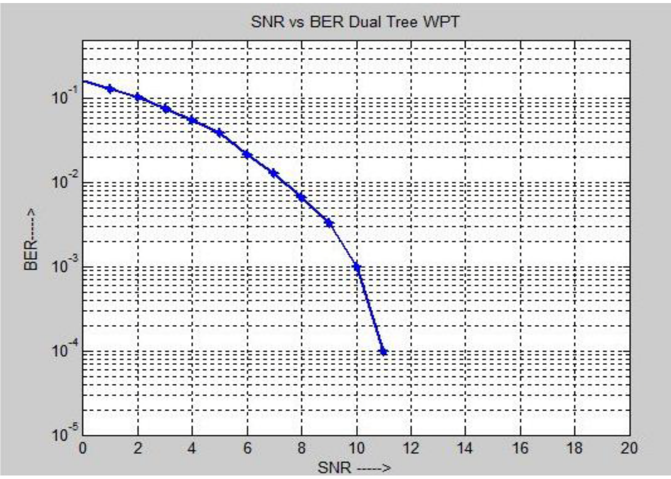

**Figure 12:** SNR vs BER for DT-WPT based OFDM System

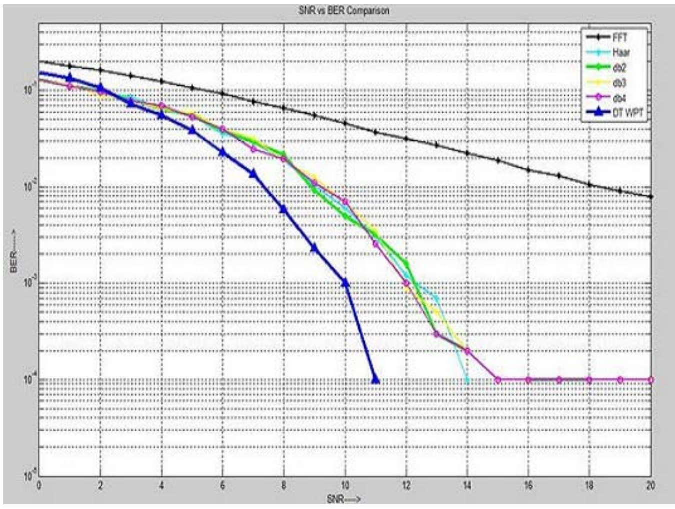

**Figure 13:** SNR vs BER comparison for all the implemented schemes

Performance of all these three schemes is compared and BER vs SNR measure is plotted over AWGN channel. In DWT-OFDM it is simulated using number of wavelet families like haar, db2, db3 and db4. Plotted comparison is shown in Fig.13. For different schemes, bit error rate BER at different signal to noise ratios SNR is given in Table I. At the SNR of 10 dB, BER for the system using DT-WPT is 0.001 which is far better than that of DWT based system and FFT based system.

**Table 1:** BER Measurements of Different Schemes

| | SNR | | | BER | | | |
|---|---|---|---|---|---|---|---|
| S. No | | FFT OFDM | DWT OFDM (Haar) | DWT OFDM (db2) | DWT OFDM (db3) | DWT OFDM (db4) | DT-WPT OFDM |
| 1. | 6 dB | 0.095 | 0.037 | 0.039 | 0.040 | 0.040 | 0.023 |
| 2. | 8 dB | 0.070 | 0.022 | 0.020 | 0.020 | 0.020 | 0.006 |
| 3. | 10 dB | 0.039 | 0.006 | 0.005 | 0.007 | 0.007 | 0.001 |

## 5. Conclusion

In this paper, we implemented three multi-carrier modulation schemes using FFT, DWT and DT-WPT. Modulation employed in all those schemes is BPSK and the performance of all the schemes are compared over the AWGN channel. We investigated DWT-OFDM system using different wavelets like haar, daubechies. Observations from the obtained results suggest that DT-WPT OFDM system outweighs the other compared schemes by satisfying the shift variance and perfect reconstruction property. Using the wavelets in OFDM, spectral efficiency can be increased since no cyclic-prefix (CP) is used. OFDM scheme using wavelets are more robust to inter symbol interference (ISI) and Inter Carrier Interference (ICI).

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